MA-UY 4423 Worksheet 3
In class, April 14, 2015

The following are problems to be worked on during class-time, with or without the collaboration of fellow students.

There is no grade for these problems, the goal is to reinforce your understanding of the material and get some programming practice in.

Problem 1: The Lorenz Attractor (re-visited)

Last class you were asked to implement a solver for the Lorenz attractor using Euler’s method. We’ll extend that during this class, but just as a reminder . . .

The following model is related to a model for atmospheric convection:

\[
\begin{align*}
  x'(t) &= \sigma (y(t) - x(t)), \\
  y'(t) &= x(t)(\rho - z(t)) - y(t), \\
  z'(t) &= x(t)y(t) - \beta z(t),
\end{align*}
\]

where \(\sigma, \rho, \beta\) are constants. The solution behaves very differently for various values of these parameters.

Solve this system again, on the interval \(t \in [0, 100]\), except this time using your own implementation of a fourth-order Runge-Kutta scheme:

\[
\begin{align*}
  q_1 &= f(t_k, y_k) \\
  q_2 &= f(t_k + \frac{h}{2}, y_k + \frac{h}{2}q_1) \\
  q_3 &= f(t_k + \frac{h}{2}, y_k + \frac{h}{2}q_2) \\
  q_4 &= f(t_k + h, y_k + hq_3) \\
  y_{k+1} &= y_k + \frac{h}{6} (q_1 + 2q_2 + 2q_3 + q_4)
\end{align*}
\]

Verify that your solver is working correctly by testing the convergence as \(h \to 0\). You can use
various sets of initial conditions and parameters, for example:

\[(x(0), y(0), z(0)) = (1, 1, 1)\]
\[\rho = 14\]
\[\sigma = 10\]
\[\beta = \frac{8}{3}\]

**Problem 2: Van der Pol oscillator**

The Van der Pol oscillator evolves as a function of time according to the following second order ODE:

\[x''(t) - \mu(1 - x^2(t))x'(t) + x(t) = 0.\]

Using the substitution \(y(t) = x'(t)\), this can be written as a system of two first-order ODEs:

\[x'(t) = y(t)\]
\[y'(t) = \mu(1 - x^2(t))y(t) - x(t).\]

Solve the above system using for \(\mu = 1\) using the standard fourth-order Runge-Kutta scheme above as well as Matlab’s own `ode45` function. Compare your results by plotting \(x(t)\) and \(y(t)\) for \(t \in [0, 20]\).

How small does your \(h\) have to be such that the solutions visually agree? What’s happening?