Methods of Applied Mathematics

Fall 2012, MATH-GA 2701.001
Tue, 1:25–3:15, room 517 WWH.

Instructor: Oliver Bühler, room 1013 WWH
obuhler@cims.nyu.edu

Prerequisites: elementary linear algebra and differential equations.

This is a first-year graduate course for all incoming PhD and Master students interested in pursuing research in Applied Mathematics.

This course provides a concise and self-contained introduction to advanced mathematical methods, especially in the asymptotic analysis of differential equations. Topics include scaling, perturbation methods, multi-scale asymptotics, Fourier transform methods, geometric wave theory, and calculus of variations.

Grading: this course will be graded as a regular course with a grade based on approximately ten homework sets.

Topics:

1) Reduction methods: dimensional analysis, scaling, similarity solutions.
2) Regular and singular perturbations, asymptotic expansions.
3) Method of multiple scales for ODEs, averaging, WKB solutions.
3) Matched asymptotic expansions, boundary layers.
4) Green’s functions, near-field and far-field expansions, multipole expansion, radiation conditions.
5) Fourier transforms methods for PDEs, stationary phase approximation and group velocity.
6) Geometric wave theory, eikonal and transport equation, inhomogeneous media, ray tracing for dispersive waves
7) Caustics for the wave equation, amplitude corrections and phase shift
8) Hamilton-Jacobi theory and calculus of variations, Hopf-Lax formula, weak solutions