Q. J. R. Meteorol. Soc. (1998), **124**, pp. 2129–2148

Permeability of the stratospheric vortex edge: On the mean mass flux due to thermally dissipating, steady, non-breaking Rossby waves

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(Received 4 November 1997; revised 29 January 1998)

SUMMARY

As part of an assessment of the flowing-processor hypothesis of Tuck *et al.* (1993) and references—see also Rosenlof *et al.* (1997)—this paper estimates possible contributions to flow through the edge of the stratospheric polar vortex due solely to distortion of the vortex by thermally dissipating Rossby waves forced from below. To isolate such contributions in a clear-cut way, and to eliminate questions about numerical dissipation and truncation error, an idealized model is studied analytically. It assumes steady conditions and non-breaking waves, the waves being stationary in some rotating frame such as that of the earth. The model is studied using two approaches: first via the generalized Lagrangian-mean formalism of Andrews and McIntyre (1978), simplified by assuming small wave amplitude a; and second via a direct consideration of the three-dimensional, finite-amplitude undulations of the vortex edge, as defined by isentropic contours of potential vorticity, avoiding the use of any mean-and-deviation formalism. It is shown, in particular, that under quasi-geostrophic scaling the Lagrangian-mean meridional velocity $\overline{v}^{\mathrm{L}}$ is given correct to $O(a^2)$ by $\overline{v}^{\mathrm{L}} \simeq -(\partial\theta_{\mathrm{B}}/\partial z)^{-1}\overline{\mathcal{H}'\partial\eta'/\partial z}$, where θ_{B} is the basic-state potential temperature, z the altitude, η' the meridional particle displacement and $\mathcal{H}'$ the wave-induced fluctuation in the diabatic rate of change of potential temperature θ. The formula for $\overline{v}^{\mathrm{L}}$ is shown to be consistent with the independently derived finite-amplitude result; and the implication of both results is that, for disturbances dissipated by infrared radiative relaxation in the wintertime lower stratosphere, $\overline{v}^{\mathrm{L}}$ may well be directed into rather than out of the vortex, though weak outward flow is possible in some cases. There is, in addition, a vertical mean flow $\overline{w}^{\mathrm{L}}$ controlled by eddy dynamics above the altitude under consideration. This is usually directed downward ($\overline{w}^{\mathrm{L}} < 0$), and can therefore push mass out of the vortex if the vortex edge has its usual upward equatorward slope. However, under typical parameter conditions for the winter stratosphere, the magnitudes are nowhere near large enough to be consistent, by themselves, with Tuck *et al.*'s statement that the vortex is 'flushed several times' during a single winter.

KEYWORDS: Dynamics Eddy transport Polar vortex Stratosphere

1. INTRODUCTION

The problem of understanding the observed midlatitude stratospheric ozone depletion (e.g. Albritton *et al.* 1995, and references) involves not only chemistry and radiation but also nontrivial, and subtle, fluid-dynamical questions. There are reasons to suppose that the edge of the stratospheric polar vortex acts as a flexible eddy-transport barrier, to some extent leaky yet strongly inhibiting the material transport associated with layerwise-two-dimensional turbulence on isentropic surfaces. The inhibition is due in part to large-scale 'Rossby-wave elasticity' and in part to horizontal shear acting at smaller scales, especially shear just outside the vortex edge (e.g. Juckes and McIntyre 1987; Norton 1994). In the lower stratosphere, such inhibition tends to isolate vortex air chemically 'primed for ozone destruction' from extravortical, midlatitude air, most of which is not thus 'primed'.

However, there has been controversy about exactly how effective the eddy-transport barrier might be. At one extreme, flow down through the vortex and out into the midlatitude lower stratosphere has been suggested as a significant factor in the chemistry of midlatitude ozone depletion. Tuck *et al.* (1993) estimate from observational data that the lower-stratospheric part of the vortex is 'flushed several times' during a single winter,

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implying, presumably, that it sustains a throughput of the order of a vortex mass or more per month, from the vortex to the midlatitude lower stratosphere. (In Rosenlof *et al.* (1997) these throughput estimates have been lowered in the light of more recent data analyses. However, even these lowered estimates would still imply that the vortex is flushed several times each winter, 70% per month, according to a referee for this paper.) If this were correct, then the vortex could act to a significant extent as a 'flowing processor', or 'flow reactor', priming large amounts of air for catalytic ozone destruction and exporting it to sunlit middle latitudes in the lower stratosphere, with implications for the ozone layer over densely populated areas.

In view of the potential seriousness of the problem there is a need to look carefully at all possibilities, even though the large throughputs estimated by Tuck *et al.* (1993) seem at first sight fluid-dynamically implausible. What exactly, we need to ask, are the possible mechanisms that could make the edge leaky, weakening the eddy-transport barrier effect, or in any way permitting a large flow through the vortex?

One mechanism, already studied extensively, is the effect of Rossby-wave breaking. This typically takes the form of 'erosion', 'peeling' or 'stripping' of filaments of air from a narrow neighbourhood of the vortex edge, and their subsequent equatorward mixing into the midlatitude stratospheric 'surf zone' by layerwise-two-dimensional turbulence on isentropic surfaces. This erosion mechanism can obviously remove some air from the vortex, helped perhaps by the effects of inertia–gravity waves (e.g. Pierce *et al.* 1994); these can break in their own manner, contributing to small-scale vertical mixing and to horizontal parcel dispersion, and hence to vortex-edge cross-diffusion and leakiness (McIntyre 1995). Tuck *et al.* (1992) had earlier suggested, from a study of aircraft data and of isentropic maps of Rossby–Ertel potential vorticity (PV) derived from meteorological analyses, that outward material transport on isentropic surfaces, presumably resulting from some such process or processes, might be significant. On the other hand, a number of investigations using numerical modelling and observational data analyses indicate typical vortex throughput rates that fall well short of a vortex mass per month. For a survey, and critical discussion, of such work see Sobel *et al.* (1997). There must always be doubts as to whether numerical and data-analytic resolutions are good enough to quantify isentropic transport processes like layerwise-two-dimensional turbulence and vortex erosion; both encompass a large range of spatial scales, some of which are almost always too fine to be well resolved (e.g. Legras and Dritschel 1993)—probably a few tens of kilometres, and possibly even kilometres in the real lower stratosphere. Such processes must inevitably be affected by the artificial diffusivities used in numerical models. But, regardless of this, we may note the obvious fact that vortex erosion at a rate of a vortex mass per month would by itself, relative to the three-month time-scale of a whole winter, amount to rapid destruction of, rather than flow through, the vortex. In other words, it is obvious that erosion, with or without the help of inertia–gravity wave breaking, cannot by itself produce a sustained flow of the required magnitude through an intact vortex (as contrasted with the 'sub-vortex', see below). Effects other than erosion, e.g. diabatic effects, would also have to be important and might even be dominant.

Diabatic heating and cooling are included, explicitly or implicitly, in many of the studies just cited. However, in order to develop insight into the significance of such effects, and into the robustness or sensitivity of any associated flow-rate estimates, we consider here an idealized, analytical model that tentatively assumes the dominance of diabatic effects, and neglects vortex erosion altogether. This requires non-breaking waves, i.e. a purely undular vortex edge; and, consistent with this, the model further assumes that conditions are strictly steady in some rotating reference frame. Recent work described in Bühler and Haynes (1998) generalizes this model to include statistically steady flows, which corroborates that our results derived here do not depend crucially on the strict steadiness of the flow. The outcome is a clean thought-experiment exhibiting the diabatic effects in pure form, i.e. a thought-experiment in which all the sideways vortex leakage is unequivocally due to diabatic effects. Furthermore, even though sharp-edged and multiple-edged vortices are included among a large range of possible cases, there are no lingering concerns about problems of numerical accuracy. The same problem was considered by McIntyre and Norton (1990), hereafter MN, (see also Nash *et al.* 1996) whose work prepared some of the theoretical ground; however, it did not go far enough to be directly applicable to the present problem of understanding vortex-edge leakage, in part because of a mistake in interpretation (see section 3 below). This comes down to a tacit, and for present purposes inappropriate, assumption that the vortex edge is vertical everywhere, an assumption that is far too restrictive, if only because it would force us to confine attention to altitude-independent undulations.

As expected from general considerations about wave–mean interaction, the present analysis confirms that diabatic effects can by themselves make the vortex edge leaky. By contrast with MNs' analysis we allow for a fully three-dimensional distortion or undulation of the vortex edge. We find that the leakage in this model can be regarded as the sum of two contributions that can usefully be distinguished from each other. The first is a mean inflow or outflow along each isentropic surface, which in this model will turn out to be controlled entirely locally in altitude, in a thought-experiment in which one prescribes the undulation of the vortex edge on the isentropic surface in question. This contribution depends on the vortex being distorted away from a zonally symmetric state by sideways undulations that are altitude-*dependent* (see (1) and (13) below). The reason is that the PV 'impermeability theorem' (Haynes and McIntyre 1990) implies, under quasi-geostrophic scaling—and this is the real significance of MNs' result—that the diabatic leakage due to local eddy correlations, averaged around the edge of an intact vortex, can be computed as if the three-dimensional eddy diabatic circulation consisted of vertical motion only, i.e. as if the horizontal (radial and azimuthal) components required by mass continuity did not exist. In order to get leakage from eddy correlations involving the vertical motion alone, the undulation (more precisely the sideways eddy displacement of the vortex edge) must have a vertical gradient. The second contribution is from mean diabatic descent across isentropes, and therefore across the vortex edge if the edge has a *mean* slope. In this model—as in a more general class of models postulating statistically steady, 'perpetual winter', conditions—such mean descent is controlled entirely by the eddy dynamics above the isentrope of interest, in the sense discussed in Haynes *et al.* (1991, 1996) under the heading 'downward control'. In the present model this corresponds to a thought-experiment in which one prescribes the undulations of material contours at all altitudes directly above the vortex edge on the isentrope of interest, as made precise by (2) below; in practice, 'all altitudes' will usually mean a few scale heights.

To derive these results two independent approaches are used here, which check each other and help to develop a feel for robustness or sensitivity. The first, presented in section 2 and appendix A, considers the small-amplitude problem within the framework of the GLM (generalized Lagrangian-mean) theory of Andrews and McIntyre (1978). This automatically provides a view of the problem in vortex-following coordinates, subject to the general caveats noted in McIntyre (1980a), and an easy route to the results. One result is a simple formula for the first, locally controlled contribution to vortex leakage, which in the GLM framework is associated with the Lagrangian-mean meridional velocity component $\overline{v}^{L}$. (This is related to the transport velocity $\overline{v}^{T}$ of Plumb and Mahlman (1987), see appendix A.) The formula, (1) below, is derived using quasi-geostrophic theory for the disturbance structure. It verifies the crucial importance of altitude-dependent vortex undulations. The second contribution, controlled from above, is associated in the GLM framework with the Lagrangian-mean vertical velocity component $\overline{w}^{\mathrm{L}}$.

The second independent approach to our results, presented in sections 3 and 4, avoids the use of any mean-and-deviation formalism. This approach also applies straightforwardly at finite disturbance amplitude, more so than the GLM approach which is applicable in principle but only at the cost of some analytical complexity (again see the caveats in McIntyre 1980a). The cross-checks between the two approaches are given in section 5 and appendix B. Section 6 gives typical order-of-magnitude estimates for the two contributions to vortex leakiness, suggesting that, as well as often being of opposite sign, each contribution has, in any case, almost certainly much smaller magnitude than a vortex mass per month.

Before turning to details, we note that it remains fluid-dynamically possible for the 'sub-vortex' region below about 400 K or 70 hPa to act as a significant flowing processor or flow reactor. The sub-vortex is not only denser than the vortex proper, but also, in the Antarctic at least, sufficiently cold for chemical priming to take place by processes like chlorine activation. Fluid-dynamically speaking, the sub-vortex is far more susceptible to mass throughput because, almost by definition, it is the polar stratospheric layer stirred by the upward-evanescent disturbances associated with tropospheric weather. Even though the large-scale wind field still looks vortex-like (as might be expected from PV inversion of the overlying PV anomaly, e.g. Robinson 1988), the sub-vortex can have its own PV anomaly significantly disrupted—or even, in principle, completely obliterated—by the stirring from below; this makes a model of the present kind inappropriate below about 400 K, and implies no fluid-dynamical difficulty in sustaining a large throughput of air and chemical constituents via inflow and outflow along isentropes. One may think of this sub-vortex transport as having the character of layerwise-two-dimensional turbulent transport, at an opposite extreme to the purely diabatic transport through an intact vortex studied, in idealized form, in the present paper. The sub-vortex problem will not be pursued here, but some further discussion from a fluid-dynamical viewpoint may be found in McIntyre (1995), and from the viewpoint of chemical evidence in Jones and Kilbane-Dawe (personal communication).

2. Lagrangian-mean meridional circulation induced by small-amplitude disturbances

Returning to the steady, purely diabatic vortex model, consider the first contribution $\overline{v}^{\mathrm{L}}$ in the idealized situation in which the dissipation of the wave-mean system is assumed to be purely thermal and the system has settled down as a whole to an exactly steady state, i.e. not only is the wave amplitude steady, but also the mean circulation. We further assume, in this first approach, that the waves involved are of small amplitude and that they approximately satisfy geostrophic and hydrostatic balance (as in quasi-geostrophic theory, though not assuming latitude-independent static stability). Under such circumstances, it follows easily from established GLM results (details in appendix A), and also from the alternative approach via transformed Eulerian-mean theory (Mo and McIntyre 1998, (5.17)), that, away from the pole, at given co-latitudinal distance r, say (r being defined as co-latitude times the earth's radius), and at given altitude z,

$$\overline{v}^{\mathrm{L}}(r, z) = -\left(\frac{\partial \theta_{\mathrm{B}}}{\partial z}\right)^{-1} \overline{\mathcal{H}' \frac{\partial \eta'}{\partial z}}, \tag{1}$$

where the sign convention for $\overline{v}^{\mathrm{L}}(r, z)$ is the same as that of $\eta'(r, z)$, the latitudinal particle displacement. We shall take both to be positive when poleward, i.e. positive for inflow and inward displacement, in deference to the standard sign convention but in the sense opposite to r. The $\partial/\partial z$ signals the importance of altitude-dependent vortex undulations already mentioned; z can be expressed in any units, for instance pressure or log pressure; note that the units cancel on the right-hand side of (1). The other symbols are $\theta_B(r, z)$, the basic-state potential temperature, and $\mathcal{H}'$, the wave-induced heating rate expressed as the fluctuation in the diabatic rate of change of potential temperature θ, not ordinary temperature T. That is, $\mathcal{H}'$ is θ_B/T_B times the wave-induced fluctuation in the diabatic contribution to the rate of change of T, with $T_B(r, z)$ the basic-state temperature; note that $\theta_B/T_B \sim 2$ to 3 in the lower stratosphere.

As the form of (1) suggests, it does not depend on any quasi-Cartesian or channel approximation but applies equally well to a polar vortex having finite radius, $r = R(z)$ say; indeed it applies at any finite radius $r \gg \eta'$ at or away from the vortex edge, though we shall be interested mainly in applying it with r taken to be the undisturbed vortex-edge position R. When the formula is thus applied, the overbar signifies the corresponding zonal average.

Equation (1) shows that $\overline{v}^L$ at a given altitude can be related entirely locally to the correlation between the disturbance diabatic heating rate $\mathcal{H}'$ and the vertical derivative of the meridional particle displacement η'. In particular, $\overline{v}^L$ is zero for disturbances that are either adiabatic ($\mathcal{H}' = 0$) or produce altitude-independent vortex-wall undulations $\partial\eta'/\partial z = 0$. Apart from the restriction to small amplitude, this last is the particular case to which the analysis of MN applies, for which they found, at finite as well as small amplitude, that 'there is no diabatic mass transport' across the vortex edge. This will be further clarified in section 3.

The second contribution $\overline{w}^L$, controlled from above in the sense already explained, is given in this model to the same (quasi-geostrophic) accuracy as (1) by the associated 'downward control integral' (Haynes *et al.*, *op. cit.*), namely:

$$\overline{w}^L(r, z) = \frac{1}{\rho_B \cos\phi} \frac{\partial}{\partial r}\left[\cos\phi \int_z^\infty \rho_B \left(\frac{\partial\theta_B}{\partial z}\right)^{-1} \overline{\mathcal{H}' \frac{\partial\eta'}{\partial z}}\bigg|_{z=\tilde{z}} \,\mathrm{d}\tilde{z}\right], \qquad (2)$$

where ϕ is latitude and $\rho_B(r, z)$ is the basic-state mass density. The units of $\overline{w}^L$ are units of z per unit time. Note the possibility that vortex leakage could be exactly zero if the vortex edge slope $|\mathrm{d}R/\mathrm{d}z| = |\overline{v}^L/\overline{w}^L|$ with appropriate signs, for instance inflow and descent, in the usual case of vortex radius increasing with altitude.

In deriving (1) and (2) we assume that the displacements η' are small; GLM theory can be used at finite amplitude, but the details are complicated and not worth pursuing here. The second approach, given in section 3, yields finite amplitude results more simply, taking direct advantage of the model assumption of steady, non-breaking waves. The upshot will be that, judiciously interpreted, (1) and (2) are qualitatively correct even at realistic amplitudes, provided always that wave breaking remains unimportant. (When it comes to order-of-magnitude estimates, however, we shall avoid using (2) beyond its role as a reminder of the downward control principle. Thus, the actual estimates will, in the end, rely only on (1) and not on (2); hence, they will rely on neglecting wave breaking only in the altitude range, say 15–25 km, to which the flowing-processor hypothesis is considered to apply.)

To obtain a more specific idea of the likely implications of (1), consider a case in which $\mathcal{H}'$ is modelled by weak Newtonian cooling, i.e.

$$\mathcal{H}' = -\alpha\theta' = -\frac{\alpha\theta_B}{g}\frac{\partial\Phi'}{\partial z}, \qquad (3)$$

where α is the Newtonian radiative cooling coefficient, θ' the isobaric potential-temperature fluctuation, Φ' the isobaric geopotential fluctuation, and g the magnitude of the acceleration due to gravity; see (A.4). For steady, quasi-geostrophic disturbances, it is readily shown (see A.7) that

$$\eta' = \Phi'/(f\overline{U}), \tag{4}$$

where f is the Coriolis parameter and $\overline{U}(r, z)$ the basic zonal flow. Substituting (3) and (4) into (1), we immediately obtain

$$\overline{v}^{\mathrm{L}} = \frac{\alpha}{gf\overline{U}}\left(\frac{\partial \ln \theta_{\mathrm{B}}}{\partial z}\right)^{-1}\left\{\overline{\left(\frac{\partial \Phi'}{\partial z}\right)^2} - \frac{1}{2\overline{U}}\frac{\partial \overline{U}}{\partial z}\frac{\partial \overline{(\Phi'^2)}}{\partial z}\right\}. \tag{5}$$

In this expression $\overline{(\ )}$ is a zonal average as before. The first term within braces is always positive, corresponding to inflow, i.e. to a poleward rather than equatorward wave-induced mass flux. This positive definiteness, corresponding to the usual sense of the 'gyroscopic pumping' of mean circulations by thermally dissipating Rossby waves, can be seen from the foregoing to be characteristic of any small amplitude quasi-geostrophic disturbance if the basic vertical shear $\partial\overline{U}/\partial z$ is zero or negligible. In all such cases the correlation in (1) is robustly negative, as is obvious from inspection of (3) and (4), when $\overline{U}$ is independent of altitude z. Included among such cases are the standard Rossby-wave solutions familiar from slow-modulation or group-velocity theory; however, there is no restriction on the horizontal shear $\partial\overline{U}/\partial r$. The results apply, in particular, to waves on a sharp-edged vortex.

If $\overline{U}$ is altitude-dependent, then the second term of (5) might not be negligible. Under normal winter stratospheric conditions, $\overline{U} > 0$ (in a frame of reference following the wave) and $\partial\overline{U}/\partial z > 0$. Then the two terms reinforce, again giving inflow, whenever the wave has a sufficiently diffractive vertical structure, such that $\overline{|\Phi'|}$ decreases upward, as with synoptic-scale weather systems penetrating the stratosphere. Otherwise, with $\overline{|\Phi'|}$ increasing upward, as often happens with disturbances of larger horizontal scale, we may expect some cancellation between the two terms. In all cases, an upper bound on possible *outflow* rates $-\overline{v}^{\mathrm{L}}$ is evidently

$$-\overline{v}^{\mathrm{L}} \leqslant \frac{\alpha}{2g}\left(\frac{\partial \ln \theta_{\mathrm{B}}}{\partial z}\right)^{-1}\left|\frac{1}{f\overline{U}^2}\frac{\partial \overline{U}}{\partial z}\frac{\partial \overline{(\Phi'^2)}}{\partial z}\right|. \tag{6}$$

This bound is unlikely to be sharp. Outflow, corresponding to positive $-\overline{v}^{\mathrm{L}}$, requires the second term in (5) to overcome the first. The ratio of the two terms satisfies the inequality

$$\left|\frac{\text{second term}}{\text{first term}}\right| \leqslant |\overline{U}|^{-1}\left|\frac{\partial \overline{U}}{\partial z}\right|\left[\overline{(\Phi')^2}\Big/\overline{\left(\frac{\partial \Phi'}{\partial z}\right)^2}\right]^{1/2}. \tag{7}$$

This is easy to prove, for instance by writing $\Phi'(r, z)$, without loss of generality, as amplitude $|\Phi'(r, z)|$ times a phase factor $\cos\{nx - \chi(r, z)\}$, for any zonal wavenumber n and function $\chi(r, z)$. The essential point is that, for any given amplitude profile $|\Phi'(r, z)|$, increasing the vertical phase tilt will increase the magnitude of the first term in (5) but not that of the second.

The possibility that the ratio of the two terms in (5) exceeds unity and weak outflow occurs is known to be realized, in at least one case of a theoretical Rossby-wave solution with sufficiently strong positive vertical shear $\partial\overline{U}/\partial z$ (Mo and McIntyre 1998). The anomalous outflow was, however, confined to a rather narrow altitude range. The well known and well observed case of 20–22 January 1992, which occurred during the European Arctic Stratospheric Ozone Experiment, is of interest here because the vortex edge was nearly vertical during a major disturbance induced by a large blocking anticyclone over Scandinavia (e.g. Norton and Carver 1994); this suggests relatively small values of $\partial\eta'/\partial z$ and, to the extent to which the Newtonian model applies, near-cancellation of the two terms in (5). Further discussion is postponed to section 6, where some numerical estimates are given.

3. Finite-amplitude disturbances

We now derive a finite-amplitude expression for the mass flux across a three-dimensional vortex edge due to steady, non-breaking, thermally dissipating Rossby waves. The finite-amplitude vortex-edge definition is now based on isentropic contours of PV, rather than material contours*. The derivation corrects and extends the first attempt on the problem discussed in MN; they did not take the vertical structure of the vortex edge properly into account. It also: (a) allows an independent check of the small-amplitude results obtained in the previous sections, bypassing the GLM formalism; and (b) allows greater confidence in our order-of-magnitude estimates for realistically large vortex-edge distortions. Indeed, the expression to be obtained is quantitatively exact in principle, though to apply it quantitatively we would need to know the precise three-dimensional shape of the vortex edge, which in turn would require elaborate numerical calculations not worth pursuing here.

For clarity it is easiest to carry out the derivation first in a polar-tangent plane approximation, and afterwards generalize to spherical geometry in section 4. We use the hydrostatic primitive equations in isentropic coordinates, in which θ is used as a vertical coordinate and x, y denote arbitrarily oriented Cartesian horizontal coordinates, though plane polars could equally well be used. The general time-dependent form of the governing hydrostatic primitive equations in isentropic coordinates is well known, see for example Andrews *et al.* (1987) or Haynes and McIntyre (1987, 1990). Important aspects of these equations include that in $xy\theta$ space isentropes appear as flat 'horizontal' planes, and that the material velocity becomes $(u, v, \mathcal{H})$, where $u = \mathrm{D}x/\mathrm{D}t$, $v = \mathrm{D}y/\mathrm{D}t$ (not to be confused with the poleward-pointing v of section 2) are the true horizontal velocity components and not components along isentropes, and where as before $\mathcal{H} = \mathrm{D}\theta/\mathrm{D}t$ is a measure of the diabatic heating.†

Observational and numerical-modelling evidence indicate that the polar vortex tends to be bounded radially by strong isentropic gradients of PV, which together with horizontal shear (Juckes and McIntyre 1987) form an 'eddy-transport barrier', strongly inhibiting turbulent eddy transport along isentropes. This suggests the use of appropriately chosen PV contours, $\Gamma(\theta)$ say, to mark the barrier region on isentropes and with it the vortex edge‡. The PV values associated with these PV contours are denoted by $Q_\Gamma(\theta)$, and may vary continuously with θ as appropriate. The result is a curved surface marking the vortex edge, and for convenience we consider its three-dimensional geometry as seen in an $xy\theta$ space, with origin at the pole (see Fig. 1).

We now consider the mass flux, $\dot{M}$ say, into the vortex through a band B of the vortex edge bounded by two isentropes θ_1 and θ_2. For steady flow as is assumed here, $\dot{M}$ is given

* The recent work in Bühler and Haynes (1998) generalizes the expression derived here to statistically steady flows, in which the circulation around the PV contour bounding the vortex edge is constant on average.

† It should also be noted that steady flow in xyz space implies steady flow in $xy\theta$ space, because the former then implies that the variable transformation to isentropic coordinates is time-independent.

‡ This assumes that the vortex edge can be marked approximately on a given isentrope by a single PV contour. We do not wish to suggest that such an edge definition would apply more generally than in the restricted problem considered here; cf. e.g. Tao and Tuck (1994).

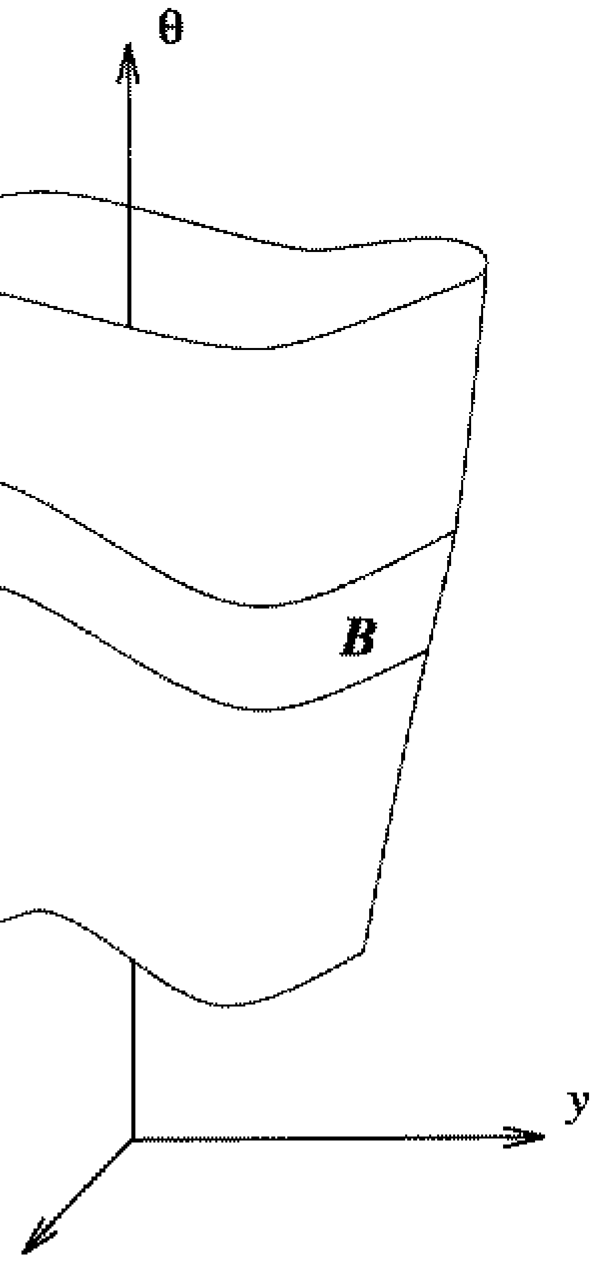

Figure 1. A schematic perspective view of the model vortex edge. Although the horizontal coordinates are labelled x, y for convenience, they need not be Cartesian (see also section 4).

by a surface integral over B as

$$\dot{M} = -\iint_{\mathrm{B}} \sigma(u, v, \mathcal{H}) \cdot \mathrm{d}\mathbf{B}, \tag{8}$$

where σ is the mass 'density' in isentropic-coordinate space such that $\sigma\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}\theta$ is the mass element, and where

$$\mathrm{d}\mathbf{B} = (\mathrm{d}y\, \mathrm{d}\theta,\ \mathrm{d}\theta\, \mathrm{d}x,\ -\mathrm{d}x\, \mathrm{d}y)$$

is an oriented surface element of B in $xy\theta$ space, pointing outwards from the vortex by definition. The dot product between the velocity vector and the surface element is simply the sum of the products of the corresponding three components (see section 4 for further discussion of this flux formula). This expression for $\dot{M}$ can be substantially simplified, because the 'horizontal' contribution to the mass flux in (8) is in fact approximately zero for steady flows in the usual quasi-geostrophic scaling regime (for large Richardson number, in a frame rotating approximately with the vortex edge).

This can be shown, following MN, by considering the PV evolution equation for steady flow in isentropic coordinates, i.e.

$$\nabla \cdot \left[\sigma Q(u, v, 0) + \mathcal{H}\left(\frac{\partial v}{\partial \theta}, -\frac{\partial u}{\partial \theta}, 0\right)\right] = 0. \tag{9}$$

Here Q is the Rossby–Ertel PV, and the symbol ∇ stands for $(\partial/\partial x,\ \partial/\partial y,\ \partial/\partial\theta)$. Note that in (9) we have again used the assumption of purely thermal dissipation. In the assumed scaling regime, namely standard quasi-geostrophic scaling as also assumed in section 2, the second term in (9) is negligible (e.g. Haynes and McIntyre 1987); therefore

$$\nabla \cdot [\sigma Q(u, v, 0)] \cong 0. \tag{10}$$

On a given isentrope (10) can be integrated over the vortex area on that isentrope, and an application of the two-dimensional divergence theorem then yields

$$\oint_{\Gamma(\theta)} \sigma Q(u, v, 0) \cdot \widehat{\mathbf{n}}\, \mathrm{d}s = Q_{\Gamma}(\theta) \oint_{\Gamma(\theta)} \sigma(u, v, 0) \cdot \widehat{\mathbf{n}}\, \mathrm{d}s \cong 0, \tag{11}$$

where $\mathrm{d}s$ is the line element along the PV contour marking the vortex edge, and where

$$\widehat{\mathbf{n}} = \left(\frac{\mathrm{d}y}{\mathrm{d}s}, -\frac{\mathrm{d}x}{\mathrm{d}s}, 0\right)$$

is the unit vector normal to the contour in $xy\theta$ space, pointing outwards by definition. In (11) $Q_{\Gamma}(\theta)$ could be moved outside the contour integral, because by assumption $\Gamma(\theta)$ is a contour of constant PV. Note that only the integral around the closed contour vanishes, not the integrand itself.

On the other hand, the horizontal contribution to the mass flux $\dot{M}$ in (8) is given by

$$-\int_{\theta_1}^{\theta_2} \oint_{\Gamma(\theta)} \sigma(u, v, 0) \cdot \widehat{\mathbf{n}}\, \mathrm{d}s\, \mathrm{d}\theta, \tag{12}$$

and comparison with (11) now shows that the inner integral is approximately zero on any given isentrope.

Therefore, in the $xy\theta$ view, only the non-horizontal part of the scalar product in (8) contributes to the mass flux into the vortex; i.e. (8) is replaced by

$$\dot{M} = \iint_{\mathrm{B}} \sigma \mathcal{H}\, \mathrm{d}x\, \mathrm{d}y, \tag{13}$$

where $\mathrm{d}x\, \mathrm{d}y$ stands for the horizontally projected area of the edge band element $\mathbf{dB}$. (Note that $\sigma\mathcal{H}$ is evaluated *on* the vortex edge band B, and not on its horizontal, i.e. xy-plane, projection.) Each area element $\mathrm{d}x\, \mathrm{d}y$ would be zero for a vortex edge that does not tilt away from the vertical in $xy\theta$ space, and the sign of $\mathrm{d}x\, \mathrm{d}y$ conforms to the convention

$$\mathrm{d}x\, \mathrm{d}y \begin{cases} > 0 : \text{edge tilts } \textit{outwards} \text{ from the vortex with increasing } \theta, \\ < 0 : \text{edge tilts } \textit{inwards} \text{ from the vortex with increasing } \theta. \end{cases} \tag{14}$$

This convention expresses the fact that diabatic heating $\mathcal{H} > 0$ produces a mass flux into the vortex wherever the vortex edge is tilting outwards with increasing θ, and *vice versa.*

Note that the result (13) has a highly nontrivial aspect; it tells us that the total diabatic mass flux into or out of the vortex through the band B can be computed as if the horizontal particle velocity components at the band could be ignored. In the assumed circumstances all such horizontal contributions, though not locally zero, sum to zero around the band. That is the significance of the partial result previously obtained in MN, described there in terms of the motion of 'quasi-material' contours on isentropic surfaces, which 'do not drift systematically northwards or southwards'. MNs' interpretation of this to mean that there is no diabatic mass transport across the vortex edge was wrong, as (13) shows, except when the vortex edge is vertical everywhere, making the right hand side of (13) zero by making the projected area elements $\mathrm{d}x\, \mathrm{d}y$ all zero. In other words, MNs' interpretation conflated what they called quasi-material transport with true material transport. Recall that the 'assumed circumstances' both here and in MN are: first, that quasi-geostrophic scaling holds, permitting neglect of the $\partial/\partial\theta$ terms in (9); and second, that all dissipative effects are diabatic. This excludes, for instance, forces attributable to gravity-wave drag.

4. GENERALIZATION TO SPHERICAL GEOMETRY AND ARBITRARY COORDINATES

Equation (13) for the mass flux $\dot{M}$ is now generalized to spherical geometry, using the straightforward spherical generalization of the tangent-plane primitive equations in isentropic coordinates (e.g. Andrews *et al.* 1987, section 3.8). This allows us to check, for instance, that the curvature of geopotentials in spherical geometry does not affect our results. It is also shown how the mass flux in spherical geometry can be expressed in terms of arbitrary coordinates (a, b, c), where (a, b) are quasi-horizontal (e.g. longitude and latitude) and c is a quasi-vertical coordinate (e.g. pressure altitude or θ). This allows us to check that the polar-tangent plane of the previous section is in fact a surprisingly accurate approximation, even for a very large polar vortex, and to do so in a succinct and economical way.

To begin with, we note that the derivation of (13) (i.e. (8) and (10) to (12)) relies only on two things: first, on defining σ such that $\sigma\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}\theta$ is the mass element; and second, on defining u and v as material rates of change of coordinate values, $u = \mathrm{D}x/\mathrm{D}t$ and $v = \mathrm{D}y/\mathrm{D}t$. With such definitions, and with (x, y, θ) replaced by (a, b, θ) (but *not* yet by (a, b, c)—see below), fluxes through surfaces (i.e. the dot product in (8) and below) and the divergence theorem can always be expressed *as if* the coordinates were Cartesian; to put it another way, the relevant equations can then all be expressed in the language of exterior calculus, without having to use the possibly complicated metric coefficients of the chosen coordinates.* This also establishes that the curvature of geopotentials does not affect our previous derivation. Hence, with the choice of $\sigma\,\mathrm{d}a\,\mathrm{d}b\,\mathrm{d}\theta$ as the mass element and with $(u, v) = (\mathrm{D}a/\mathrm{D}t, \mathrm{D}b/\mathrm{D}t)$ as quasi-horizontal velocity components, the above derivation is valid for any pair of quasi-horizontal coordinates a and b.

We finally generalize to an arbitrary quasi-vertical coordinate c by noting that the projection of the surface element in (13) is in fact independent of the choice of this quasi-vertical coordinate. This means that (13) is transformed into arbitrary coordinates (a, b, c) on the sphere by simply replacing $\mathrm{d}x\,\mathrm{d}y$ with $\mathrm{d}a\,\mathrm{d}b$, and by replacing σ with an expression derived from

$$\mathrm{d}m = \sigma\,\mathrm{d}a\,\mathrm{d}b\,\mathrm{d}\theta = \rho\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z \Leftrightarrow \sigma = \rho\frac{\partial(x, y, z)}{\partial(a, b, c)}\left(\frac{\partial\theta}{\partial c}\right)^{-1}, \tag{15}$$

where ρ is the mass density in ordinary, geometric space, and $\partial\theta/\partial c$ is evaluated at constant a and b. Therefore $\dot{M}$ is re-expressed in arbitrary coordinates (a, b, c) on the sphere as

$$\dot{M} = \iint_{\mathrm{B}} \rho\frac{\partial(x, y, z)}{\partial(a, b, c)}\left(\frac{\partial\theta}{\partial c}\right)^{-1} \mathcal{H}\,\mathrm{d}a\,\mathrm{d}b, \tag{16}$$

where B stands for the image of the vortex band in the chosen coordinates and where $\mathrm{d}a\,\mathrm{d}b$ conforms to the sign convention in (14).

In local Cartesian coordinates $(a, b, c) = (x, y, z)$, which are suitable for a tangent-plane approximation, this is simply

$$\dot{M} = \iint_{\mathrm{B}} \rho\left(\frac{\partial\theta}{\partial z}\right)^{-1} \mathcal{H}\,\mathrm{d}x\,\mathrm{d}y. \tag{17}$$

Alternatively, if longitude λ, co-latitudinal distance r, and altitude z are used as coordinates

* In this notation the diabatic flux in the full PV evolution (9) looks more complicated on the sphere than on a tangent plane (cf. Andrews *et al.* 1987, Eq. (3.8.8)). However, as argued before, the diabatic flux is negligible in the assumed scaling regime.

(a, b, c), then $\dot{M}$ is given by

$$\dot{M} = \iint_{\mathrm{B}} \rho \frac{(z + a_{\mathrm{E}})^2}{a_{\mathrm{E}}} \sin(r/a_{\mathrm{E}}) \left(\frac{\partial \theta}{\partial z}\right)^{-1} \mathcal{H} \, \mathrm{d}\lambda \, \mathrm{d}r, \approx \iint_{\mathrm{B}} \rho r \left(\frac{\partial \theta}{\partial z}\right)^{-1} \mathcal{H} \, \mathrm{d}\lambda \, \mathrm{d}r, \qquad (18)$$

where a_{E} is the earth's radius such that the ratio r/a_{E} is equal to co-latitude. The second, approximate form comes from noting that $z \ll a_{\mathrm{E}}$ and that $a_{\mathrm{E}} \sin(r/a_{\mathrm{E}}) \approx r$ for realistic values of co-latitudinal distance for the vortex edge, i.e. for values of r not much larger than $a_{\mathrm{E}}/2$, say. (See section 6 for numerical estimates of r.) The second form shows that for realistic values of r the mass flux $\dot{M}$ can, to a very good approximation, be calculated using cylindrical polar coordinates based on a polar-tangent plane, in which co-latitudinal distance r acts as radial distance. This polar-tangent-plane approximation can equally well be described using the local Cartesian coordinates introduced in (17), provided that x and y are related to the co-latitudinal distance r by $r^2 = x^2 + y^2$. This very useful description of the mass flux $\dot{M}$ will be used repeatedly below.

5. Connection to small-amplitude theory

To derive a leading-order estimate for (17) and check that it agrees with the previous small-amplitude result, we assume that the flow consists of three components: first, a zonally symmetric basic state whose meridional velocity components are zero; second, a field of small-amplitude linear Rossby waves with non-dimensional amplitude $a \ll 1$; and third, an $O(a^2)$ mean-flow response to these linear Rossby waves. This implies that the entire mean meridional circulation is wave-driven and $O(a^2)$. It also implies that the undisturbed, basic-state polar vortex is zonally symmetric, i.e. that its bounding PV contours are latitude circles. The shape of the basic-state vortex edge, B_0 say, can then be described by an altitude-dependent radius $R(z)$ in cylindrical polar coordinates with origin at the pole—either in tangent-plane or in spherical polar coordinates, see (17) and (18). The slope of the basic-state vortex edge is given by $\mathrm{d}R/\mathrm{d}z$, which in the real winter stratosphere is often observed to be a modest fraction of $N/f \gg 1$, which latter $\gtrsim 10^2$. Such a substantial slope of the basic-state vortex edge cannot be neglected *a priori* in its contributions to the projected surface elements $\mathrm{d}x \, \mathrm{d}y$ in (17).

This zonally-symmetric basic-state vortex edge B_0 corresponds to the leading-order mean vortex edge in the GLM formalism. This implies that the shape of the actual vortex edge, which is undulated by the Rossby waves, is given to leading order by the map $\mathbf{x} \mapsto \mathbf{x} + \boldsymbol{\xi}'(\mathbf{x}, t)$, for all $\mathbf{x}$ on B_0. Here $\boldsymbol{\xi}'$ is the $O(a)$ linearized GLM particle displacement field used previously. This map from B_0 to B allows us to write the integral in (17) as an integral over B_0 whilst evaluating the integrand on B. The leading-order result of this procedure, whose details are given in appendix B, is

$$\dot{M} = 2\pi \int_{z_1}^{z_2} \rho_{\mathrm{B}} \left(\frac{\partial \theta_{\mathrm{B}}}{\partial z}\right)^{-1} \left[-\overline{\left(\mathcal{H}' \frac{\partial \eta'}{\partial z}\right)} + \overline{\mathcal{H}}^{\mathrm{L}} \frac{\mathrm{d}R}{\mathrm{d}z} \right] R \, \mathrm{d}z + O(a^3), \qquad (19)$$

where $z = z_1$ and $z = z_2$ are the horizontal planes bounding the edge band B_0, and where $\overline{\mathcal{H}}^{\mathrm{L}}$ is evaluated correct to $O(a^2)$; η' is, as in previous sections, the *poleward* particle displacement correct to $O(a)$, which accounts for the minus sign in the integrand. (The sign convention is consistent when $\overline{v}^{\mathrm{L}}$ is poleward velocity and $\dot{M}$ inward mass flux, as before.) Note that ρ and $\partial\theta/\partial z$ have been replaced by their zonally symmetric basic-state values ρ_{B} and $\partial\theta_{\mathrm{B}}/\partial z$.

The two terms in the square brackets are the leading-order expressions for the two contributions to the mean transport discussed earlier. The first contribution $-\overline{(\mathcal{H}'\partial\eta'/\partial z)}$ describes the mean transport across the vortex edge along isentropes, which is given by the eddy structure in the local altitude range $z_1 < z < z_2$, specifically, by the local correlation between disturbance-induced diabatic heating and vortex-edge tilt. The form of this first term gives an independent check on the small-amplitude expression for $\overline{v}^{\mathrm{L}}$ presented in (1), in particular checking the previous assertion that the validity of the expression does not depend on any quasi-Cartesian or beta-channel approximation; there are no extra terms in $1/R$, for instance. The second contribution $\overline{\mathcal{H}}^{\mathrm{L}}\mathrm{d}R/\mathrm{d}z$ describes the mean transport across isentropes, corresponding to $\overline{w}^{\mathrm{L}}$ in (2).

6. Numerical orders of magnitude

We now use (19) to obtain some typical order-of-magnitude estimates for the two contributions to the mass flux, with realistic values for the diabatic heating and wave amplitude. Although finite-amplitude analogues of (19) can be derived in principle from (17), by using suitably defined finite-amplitude formulae for the edge projections $\mathrm{d}x\,\mathrm{d}y$ and so on, it is clear from the analysis just sketched (given in detail in appendix B) and from the associated geometric picture, that (19) is robust enough to be used for order-of-magnitude estimates.

The ratio between the vortex mass M and the mass transport rate $\dot{M}$ into or out of the vortex may be called the vortex flushing time, τ_{F} say, i.e.

$$\tau_{\mathrm{F}} \equiv -M/\dot{M}, \tag{20}$$

which is positive for outflow if $\dot{M} < 0$ corresponds to outflow. In a steady state ($M/\dot{M}$ constant), τ_{F} is the time required for a throughput of just 1 vortex mass. Note also that τ_{F} allows us to define unambiguously the relative vortex-mass throughput per unit time in our thought-experiment. We assume that the vortex edge is sufficiently well-defined, and undular, from $\theta = 400$ K upward to use the corresponding altitude z_1 as the lower boundary of the vortex edge in our model. The vortex itself is assumed to extend over several (3–5, say) density-scale heights $H \sim 7$ km in the vertical. The rapid decay of the basic state mass density $\rho_{\mathrm{B}}(r, z)$ with altitude, which tends to make high-altitude contributions to M unimportant, then allows us to treat the vortex height as effectively infinite for the purpose of calculating M. For the purpose of estimating the first contribution to (19), that involving $\partial\eta'/\partial z$, the geopotential disturbance Φ' is assumed to have a simple exponential vertical structure without phase tilt. The no-phase-tilt assumption is not essential, but is made here because non-zero phase tilt always increases the inflow component of this contribution (noted already below (7)). The second contribution, that involving $\overline{\mathcal{H}}^{\mathrm{L}}$, will be estimated without direct reference to the types of disturbances that might be acting to pump air down from above, but simply by using observational estimates of typical descent and cooling rates. We look at this next.

Consider, then, the second contribution in (19), i.e. the transport due to mean diabatic heating $\overline{\mathcal{H}}^{\mathrm{L}}$ in the presence of a sloping mean vortex edge. We assume that $\overline{\mathcal{H}}^{\mathrm{L}}$ is constant and equal to a typical negative value, compatible with studies of mean descent in the polar vortex (e.g. Manney *et al.* 1994). The mean vortex-edge slope $\mathrm{d}R/\mathrm{d}z$ is assumed to be a positive constant, γ say, whose value is a moderate fraction of N/f. This is consistent with typical observations. Together these assumptions imply that the second contribution corresponds to $\dot{M} < 0$, i.e. to mean outflow. The vortex mass is estimated as

$$M \approx \int_{z_1}^{+\infty} \pi \rho_B R^2 \, dz = \pi \rho_1 R_1^2 H \left(1 + 2\frac{\gamma H}{R_1} + 2\frac{\gamma^2 H^2}{R_1^2}\right), \tag{21}$$

where ρ_1 and R_1 are the density and vortex radius at the lower boundary, $\rho_B = \rho_1 \exp(-(z - z_1)/H)$, and $R = R_1 + \gamma(z - z_1)$. The second or $\overline{\mathcal{H}}^L$ contribution to (19) is found to be

$$\dot{M} \approx 2\pi \int_{z_1}^{+\infty} \rho_B \left(\frac{\partial \theta_B}{\partial z}\right)^{-1} \overline{\mathcal{H}}^L \gamma R(z) \, dz = 2\pi \rho_1 \overline{\mathcal{H}}^L \gamma \frac{H^2 R_1}{\kappa(\kappa+1)\theta_1} \left(1 + \frac{\gamma H}{R_1(\kappa+1)}\right), \tag{22}$$

where $\theta_B \approx \theta_1 \exp(\kappa(z - z_1)/H)$ has been assumed, with κ = (gas constant)/(constant-pressure specific heat) = 0.286. Combining (22) and (21) we obtain

$$\tau_F \approx -\frac{\kappa(\kappa+1)\theta_1 R_1}{2\gamma \overline{\mathcal{H}}^L H} \frac{1 + 2\dfrac{\gamma H}{R_1} + 2\dfrac{\gamma^2 H^2}{R_1^2}}{1 + \dfrac{\gamma H}{R_1(\kappa+1)}}. \tag{23}$$

Using $\overline{\mathcal{H}}^L = -0.6$ K day^{-1} in θ (~ -0.3 K day^{-1} in T; e.g. Manney, *op. cit.*), $\theta_1 = 400$ K, $\gamma = 0.5N/f \approx 80$, $H = 7$ km, and $R_1 = 3000$ km (which corresponds to a mean vortex edge bounded on $\theta = 400$ K by 63° latitude) then gives $\tau_F \approx 850$ days. This shows that the mean outflow due to diabatic descent across the sloping vortex edge is weak; weaker even than the mean outflow through the vortex bottom into the 'sub-vortex' below $\theta = 400$ K (cf. McIntyre 1995, §6; Jones and Kilbane-Dawe personal communication), which by itself can be estimated as $\dot{M} \approx \pi R_1^2 \rho_1 \overline{\mathcal{H}}^L H/(\kappa\theta_1)$, yielding a shorter flushing time $\tau_F \approx 270$ days.

Consider finally the first or $\mathcal{H}'$ contribution to (19), i.e. the along-isentrope transport due to a correlation between fluctuations $\partial\eta'/\partial z$ in vortex-edge tilt and fluctuations $\mathcal{H}'$ in diabatic heating rate, corresponding to $\overline{v}^L$ in (1). This equation was rewritten in terms of geopotential disturbance Φ' in (5) for the case of small-amplitude Rossby waves dissipated by Newtonian cooling in the wintertime lower stratosphere. It is reasonable to expect that such a relation also applies approximately to finite-amplitude undulations of the vortex edge whose temperature anomalies suffer radiative relaxation, and whose phase relations are qualitatively those implied by geostrophic and hydrostatic balance. Hence, for the purpose of order-of-magnitude estimates, we assume that the relations between $\mathcal{H}'$, η' and Φ' implied by the small-amplitude (3) and (4), namely

$$\mathcal{H}' = -\alpha\theta' = -\alpha \frac{\theta_B}{g} \frac{\partial \Phi'}{\partial z} \quad \text{and} \quad \eta' = \frac{\Phi'}{f\overline{U}}, \tag{24}$$

also hold approximately at finite amplitude. Here α is a suitably chosen Newtonian radiative cooling coefficient.

We assume a simple form of Φ' on the vortex edge without vertical phase tilt, namely

$$\Phi' = \Phi_1' \cos(kx) \exp(-(z - z_1)/h), \tag{25}$$

where Φ_1' is the geopotential-disturbance amplitude at the vortex bottom, k is a zonal wavenumber, and h is the exponential envelope scale. Both decaying ($h > 0$) and growing ($h < 0$) disturbances are considered. We now restrict attention to the lower part of the vortex, say the lowest 10 km or so, i.e. $D \equiv z_2 - z_1 = 10$ km. This avoids non-essential convergence problems of the integral in (19) if the disturbances are growing very strongly with altitude; see also the Concluding Remarks. Non-zero vertical shear must be explicitly recognized, as in (5), and the along-isentrope mass flux in (19) is then estimated from (24) and (25) as

$$\dot{M} \approx \pi\alpha\rho_1 R_1^2 \mathrm{Ro}\frac{f^2}{N^2}\frac{\eta_1'^2}{h^2}\left[1+\frac{h}{\overline{U}}\frac{\partial\overline{U}}{\partial z}\right]\int_0^D \exp(-z/\lambda)\left(1+\frac{\gamma z}{R_1}\right)\,\mathrm{d}z \tag{26}$$

where the effective exponential envelope scale λ, which includes density decay as well as the amplitude envelope of Φ'^2, is given by

$$\lambda \equiv \left(\frac{1}{H}+\frac{2}{h}\right)^{-1}. \tag{27}$$

This uses the same assumptions for ρ_{B} etc. as before, $\mathrm{Ro} \equiv \overline{U}_1/(fR_1)$ is the Rossby number at the vortex bottom, η_1' is the vortex undulation amplitude at the bottom, and $\overline{U}(z)$ has been approximated with $\overline{U}_1$ except in the explicit shear term, which is itself approximated as a constant. Note that the square bracket determines the sign of $\dot{M}$, and that this bracket exhibits the previously mentioned competition between inflow and outflow contributions in the case of $h<0$ and $\overline{U}^{-1}\partial\overline{U}/\partial z>0$.

The integral in (26) can be evaluated as

$$\int_0^D \exp(-z/\lambda)\left(1+\frac{\gamma z}{R_1}\right)\,\mathrm{d}z = \lambda\left\{1+\lambda\frac{\gamma}{R_1}-\exp(-D/\lambda)\left(1+\frac{\gamma}{R_1}(\lambda+D)\right)\right\} \tag{28}$$

and hence the corresponding flushing time is estimated as the ratio of M from (21) and $-\dot{M}$ from (26), (27) and (20). Let $\alpha = 0.05$ day^{-1}, $\mathrm{Ro} = 0.1$ (which corresponds to $\overline{U}_1 \approx 40$ m s^{-1}), $D = 10$ km, $\eta_1' = 1500$ km, and let the remaining parameters take the values previously used. Consider first a case without vertical shear (i.e. $\partial\overline{U}/\partial z = 0$) and with a decaying geopotential-disturbance structure described by $h = 10$ km. This yields $\tau_{\mathrm{F}} \approx -700$ days, where it should be noted that the sense of this contribution is *into* the vortex. Second, still without vertical shear, let the geopotential disturbance be growing with $h = -10$ km. This implies much stronger flow yielding $\tau_{\mathrm{F}} \approx -140$ days, but this is still a contribution into the vortex. Finally, consider strong vertical shear such that $\overline{U}(\partial\overline{U}/\partial z)^{-1} = 10$ km. (This is consistent with observed values of this vertical shear length-scale, which typically range from 10–30 km in the lower stratosphere.) It is then found that maximal outflow (i.e. minimal positive τ_{F}) is achieved when $h \approx -16$ km, yielding $\tau_{\mathrm{F}} \approx 880$ days.

It can be noted that the positive mean slope of the vortex γ reduces the effectiveness of the along-isentrope contributions, because the mass of the vortex increases as R^2 whereas the strength of the flow along isentropes increases only as R. We have experimented considerably with different choices for the various parameters that enter these estimates, but have found no indication that the flushing times can be brought close to the times required for the flowing-processor hypothesis mentioned before.

7. Concluding Remarks

Analytical formulae, and order-of-magnitude estimates, for the sideways ($\overline{v}^{\mathrm{L}}$ associated) and vertical ($\overline{w}^{\mathrm{L}}$ associated) contributions to the mean mass flux through the edge of an intact polar stratospheric vortex have been presented. The notion of intactness is expressed theoretically by the basic assumption made here, that vortex erosion and other effects of Rossby-wave breaking can be neglected in estimating $\overline{v}^{\mathrm{L}}$ and the associated sideways mass flux through the lower-stratospheric part of the vortex, say a 10 km-thick layer above 15–16 km. The $\overline{w}^{\mathrm{L}}$ contribution, by contrast, is estimated on the assumption that it is pumped from above by eddy activity, which might, and probably does, include significant wave breaking higher up. All the mass-flux contributions appear to be too small to support the version of the flowing-processor hypothesis that postulates large flows of the order of a vortex mass per month through the polar vortex, as distinct from the sub-vortex below about 400 K, to which the notion of intactness does not apply and through which large flows could much more easily take place.

Contrary to what the simplest wave–mean theories suggest, the sense of the $\overline{v}^{\mathrm{L}}$ contribution can be either into or out of the vortex when the background zonal wind has sufficient vertical shear, as illustrated by (5). There are then intermediate cases in which the $\overline{v}^{\mathrm{L}}$ contribution vanishes even though $\partial\Phi'/\partial z$ and $\mathcal{H}'$ do not, of which the most obvious are cases in which the vortex wall is distorted yet vertical everywhere—as was approximately true, for instance, in a case analysed by Norton and Carver (1994). Indeed, in such cases there must be zero net mass flux through the edge, as can be seen from the finite-amplitude formula (13); note further that refinements to the theory, such as going beyond quasi-geostrophic accuracy, will not qualitatively change the picture beyond changing the stipulation 'vertical' to 'slightly sloping' (by a small fraction of Prandtl's ratio N/f). As discussed for instance in MN and in Mo and McIntyre (1998), this does not, of course, imply that other measures of mean circulation need vanish.

The assumption of negligible wave breaking is critical only as regards conditions in the vortex edge, where material contours undulate more or less reversibly and quantities like $\overline{v}^{\mathrm{L}}$ are well defined. The assumption probably represents a good approximation if, as seems typical in the lower stratosphere, the actual wave breaking gives rise to no more than the erosion of fine filaments or sheets from the vortex edge. Such erosion will not substantially change typical θ' anomalies and hence will not produce substantial diabatic effects, because PV inversion tends to have the character of a smoothing operator and hardly sees the finer-scale structures. In other words, it takes a large-scale distortion of the vortex to generate substantial θ' anomalies and eddy diabatic effects. For order-of-magnitude purposes, such large-scale distortion is probably described well enough by a theory of the kind used above, in which the theoretical displacement field η' is interpreted as, in effect, a measure of the large-scale distortion only, and not as the exact, and irreversible, material distortion including wave breaking effects in the form of fine-scale erosion. In this sense, linearized wave–mean theory is 'better', for present purposes, than nonlinear theory. Better still, however, is the finite-amplitude formula (18), which avoids the problems of finite-amplitude wave–mean theory and is valid for any intact vortex. This latter would be the best of the foregoing results to use if the theory were to be applied in an observational case study.

One might ask about the fate of outflow from the side of the vortex, as distinct from outflow from the bottom, as well as asking about horizontal flow through the sub-vortex. On present evidence concerning vertical, cross-isentropic diffusivities in the lower stratosphere (tending to confirm that they can be taken to be of the order of 0.2 $\mathrm{m^2s^{-1}}$ as usually supposed, e.g. most recently in Sparling *et al.* 1998 and references) we can argue that vertical diffusion heights would be little more than a kilometre, over the time-scale of one winter, implying that vortex and sub-vortex outflows would tend to have separate midlatitude destinations at separate altitudes within the layer of interest for midlatitude ozone depletion chemistry, which we have been assuming to lie between about 15 km and

25 km. This suggests that flow out from the bottom of the vortex should be considered as part of the sub-vortex problem, significant for chemistry in the midlatitude stratosphere below about 15 km as long as mean descent does not take it out of the stratosphere altogether; while flow out of, or into, the sides of the vortex should be considered separately, and is significant for chemistry in the midlatitude stratosphere between about 15 km and 25 km.

Although, as stated at the outset, the importance of the ozone-depletion problem requires us to leave no stone unturned, the analysis has brought no surprises beyond finding that weak outflow $\overline{v}^{\mathrm{L}}$ is a possibility, in some rather restricted circumstances. This possibility was not revealed by earlier simplistic arguments in terms of angular momentum (McIntyre 1995), which fail to allow for the diabatic distortion of material contours; though such arguments, as it has turned out, still give a correct idea of the likely order of magnitude of $\overline{v}^{\mathrm{L}}$. Consistent with our present arguments about robustness, the estimates made here are not much larger than the similarly modest vortex throughput rates derived from the numerical-model studies; again see the critical discussion in Sobel *et al.* (1997) and references. We conclude that it is difficult to find any fluid-dynamical justification for the suggestion of high throughputs of the order of a vortex mass per month. We think our estimates are robust enough to permit some confidence that the only way to get such throughputs would be for cooling time-scales in the lower stratosphere to be more like 5 days than 20 days, which seems unlikely.

Acknowledgements

We thank P. H. Haynes, R. L. Jones, I. Kilbane-Dawe, W. A. Norton, J. A. Pyle and T. G. Shepherd for stimulating correspondence and conversations, and two referees for constructive and encouraging comments. The result (11) was first pointed out to us by P. H. Haynes. RM gratefully acknowledges support by the British Council and the State Education Commission of the People's Republic of China in form of a 4-year Sino–British Friendship Scholarship. OB thanks the Gottlieb Daimler and Karl Benz foundation in Germany and the Natural Environment Research Council for research studentships. MEM thanks the Engineering and Physical Sciences Research Council for generous support in the form of a senior research fellowship.

Appendix A

Derivation of (1) and remarks on $\overline{v}^{\mathrm{T}}$

Equation (1) can be derived from theorem I and corollary II of Andrews and McIntyre (1978); see also McIntyre (1980b), Eqs. (4.10b), (5.4a), (5.5) and (5.7) which are essentially the same results in a slightly more convenient form for our purpose. It is easiest to begin by thinking in terms of a quasi-Cartesian or beta-channel approximation, though it can be shown from a version of corollary II (and can be independently seen from section 5 above) that the result, in the end, depends on no such approximation.

In the present steady-state problem, with small wave amplitude and purely thermal wave dissipation and no significant wave breaking, the results just mentioned imply at once that

$$f\overline{v}^{\mathrm{L}} = -\frac{g}{\theta_{\mathrm{B}}}\overline{\theta^{\ell}\frac{\partial \zeta'}{\partial x}} + O(a^3) = \frac{g}{\theta_{\mathrm{B}}}\overline{\zeta'\frac{\partial \theta^{\ell}}{\partial x}} + O(a^3), \tag{A.1}$$

where $\overline{(\ )}$ is a zonal average as before, $\overline{v}^{\mathrm{L}}$ is the GLM meridional velocity, f is the Coriolis parameter, g is the magnitude of gravity acceleration, $\theta\mathrm{B}$ is the basic state potential temperature, ζ' is the upward particle displacement, a is a measure of the eddy amplitude, and θ^ℓ is the Lagrangian potential-temperature fluctuation given to leading order by

$$\theta^\ell = \theta' + \eta' \frac{\partial \theta_B}{\partial y} + \zeta' \frac{\partial \theta_B}{\partial z} + O(a^2). \tag{A.2}$$

As before, η' is the northward particle displacement. (A.2) may be rearranged as

$$\zeta' = \left(\theta^\ell - \theta' - \eta' \frac{\partial \theta_B}{\partial y}\right) \left(\frac{\partial \theta_B}{\partial z}\right)^{-1}. \tag{A.3}$$

From hydrostatic balance we have, with Φ' the disturbance and $\overline{\Phi}$ the mean geopotential,

$$\frac{g\theta'}{\theta_B} = \frac{\partial \Phi'}{\partial z}, \quad \frac{g}{\theta_B}\frac{\partial \theta_B}{\partial y} = \frac{\partial^2 \overline{\Phi}}{\partial y \partial z} = -f \frac{\partial \overline{U}}{\partial z}, \tag{A.4}$$

because

$$\frac{\partial \overline{\Phi}}{\partial y} = -f\overline{U}. \tag{A.5}$$

Now invoking geostrophic balance, steadiness, and small wave amplitude again, we see that, to leading order, Φ' is related to η' through

$$\frac{1}{f}\frac{\partial \Phi'}{\partial x} = v' = v^\ell = \overline{U}\frac{\partial \eta'}{\partial x}, \tag{A.6}$$

whence

$$\Phi' = f\overline{U}\eta', \tag{A.7}$$

equivalent to (4). Similarly

$$\overline{U}\frac{\partial \theta^\ell}{\partial x} = \mathcal{H}', \text{ i.e. } \frac{\partial \theta^\ell}{\partial x} = \frac{\mathcal{H}'}{\overline{U}}, \tag{A.8}$$

where $\mathcal{H}'$ is the wave-induced fluctuation in the diabatic rate of change of θ, and $\overline{U}$ is the basic zonal flow. Substituting (A.3) into (A.1) and using (A.7) and then (A.4) and (A.8) gives (1).

We remark that, as shown by Plumb (1979) within the quasi-Cartesian or beta-channel approximation, $(\overline{v}^L, \overline{w}^L) = (\overline{v}^T, \overline{w}^T)$ where $(\overline{v}^T, \overline{w}^T)$ denotes the effective transport circulation in the sense of Plumb (1979) and Plumb and Mahlman (1987). An alternative derivation is given in Mo and McIntyre (1998), also within the channel approximation. Both derivations depend on the assumption of steady, non-breaking waves. Whether the result, unlike (1), depends on the channel approximation in any essential way is not clear at present, because the theory of the effective transport circulation does not seem to have been fully worked out in curvilinear geometry. For instance, the Stokes drift contribution to $(\overline{v}^L, \overline{w}^L)$ in polar-cap or other curvilinear geometry includes a term involving the second spatial derivatives of the basic zonal velocity field. These derivatives form a third-rank tensor some of whose components are $O(1)$ as distinct from $O(a^2)$, hence not negligible, for disturbances of low zonal wavenumber when latitude circles are significantly curved, and when the zonal velocity is $O(1)$. For theoretical detail the reader may consult Andrews and McIntyre (1978). For instance the second term in their (2.27), with the factor $\overline{\varphi}_{,ij}$ interpreted as representing the second derivatives of the basic zonal velocity field, with one of the three indices implicit, contains a significant contribution proportional to the correlation $\frac{1}{2}\overline{\xi'\eta'}$ where ξ' is the zonal disturbance displacement. To see whether similar terms arise when generalizing the formulae for $(\overline{v}^{\mathrm{T}}, \overline{w}^{\mathrm{T}})$ to a significantly curved geometry, one would have to develop a model for the associated three-dimensional diffusion tensor that is sensitive to the presence of $O(1)$ mean shear and mean-flow curvature, as might possibly happen through shear-dispersion effects.

We recall finally that, even in the channel approximation, $(\overline{v}^{\mathrm{L}}, \overline{w}^{\mathrm{L}})$ and $(\overline{v}^{\mathrm{T}}, \overline{w}^{\mathrm{T}})$ differ from the transformed Eulerian-mean meridional velocity $(\overline{v}^{*}, \overline{w}^{*})$. Mo and McIntyre (1998) present a specific example in which they have the opposite sign.

Appendix B

Derivation of (19)

The task is to derive (19) as the $O(a^2)$ approximation of (17) in the case of an axisymmetric basic-state polar vortex perturbed by $O(a)$ Rossby waves. The integral over the actual vortex edge B in (17) is replaced by an integral over the leading-order mean vortex edge, i.e. the axisymmetric basic-state vortex edge B_0. The diabatic heating $\mathcal{H}$ then has to be evaluated at the displaced position $\mathbf{x} + \boldsymbol{\xi}'$, and this gives

$$\mathcal{H}(\mathbf{x} + \boldsymbol{\xi}') = \mathcal{H}' + \overline{\mathcal{H}} + \boldsymbol{\xi}' \cdot \nabla \mathcal{H}' + O(a^3) \tag{B.1}$$

correct to $O(a^2)$, where the terms on the right-hand side are all evaluated at $\mathbf{x}$. Specifically, $\overline{\mathcal{H}}$ and $\mathcal{H}'$ denote the Eulerian mean and disturbance parts of the diabatic heating $\mathcal{H}$, where the mean is a zonal average as before (cf. (B.7) below). Because the diabatic heating $\mathcal{H}$ is itself $O(a)$, all remaining terms in the integrand of (17) need only be evaluated correct to $O(a)$ in order to yield all contributions at $O(a^2)$.

The GLM formalism provides a general relation between a disturbed surface area element $\mathbf{dB}$ and the corresponding mean surface area element $\mathbf{dB}_0$ (Andrews and McIntyre 1978, section A.3). The leading-order form of this general relation is

$$\mathbf{dB} = \mathbf{dB}_0(1 + \nabla \cdot \boldsymbol{\xi}') - \mathbf{dB}_0 \cdot (\nabla \boldsymbol{\xi}')^{\mathrm{T}} + O(a^2), \tag{B.2}$$

where $\mathbf{dB}_0 \cdot (\nabla \boldsymbol{\xi}')^{\mathrm{T}}$ is equal to $[\mathbf{dB}_0]_j \xi'_{j,i}$ when written in Cartesian coordinates. For the axisymmetric basic-state vortex edge the surface area element and the $O(a)$ displacement field $\boldsymbol{\xi}'$ are given by, respectively,

$$\mathbf{dB}_0 = \left(\widehat{\mathbf{r}} - \frac{\mathrm{d}R}{\mathrm{d}z}\widehat{\mathbf{z}}\right) \mathrm{d}s\, \mathrm{d}z \text{ and} \tag{B.3}$$

$$\boldsymbol{\xi}' = -\eta'\widehat{\mathbf{r}} + \xi'\widehat{\mathbf{t}} + \zeta'\widehat{\mathbf{z}}. \tag{B.4}$$

The function $R(z)$ gives the radius of the basic-state vortex edge in cylindrical polar coordinates and $(\widehat{\mathbf{r}}, \widehat{\mathbf{t}}, \widehat{\mathbf{z}})$ are local unit vectors in these coordinates, pointing in the radial, azimuthal, and vertical directions respectively. The increment $\mathrm{d}s$ is taken along latitude circles. Note that the components of the displacement vector have been chosen according to the convention that (ξ', η', ζ') are the Cartesian components of the displacement vector with respect to a local tangent plane oriented such that ξ' points in the zonal direction and η' points poleward.

Using (B.3) and (B.4) in (B.2) allows the horizontal projections of **dB** to be evaluated as

$$\begin{aligned} \mathrm{d}x\,\mathrm{d}y &= -\mathbf{dB}\cdot\widehat{\mathbf{z}} \quad \text{(by chosen sign convention for } \mathrm{d}x\,\mathrm{d}y \text{ in (14))} \\ &= \frac{\mathrm{d}R}{\mathrm{d}z}(1+\nabla\cdot\boldsymbol{\xi}'-\zeta_z')\,\mathrm{d}s\,\mathrm{d}z - \eta_z'\,\mathrm{d}s\,\mathrm{d}z + O(a^2) \\ &= \left(\frac{\mathrm{d}R}{\mathrm{d}z}-\eta_z'\right)\mathrm{d}s\,\mathrm{d}z + O(a^2), \end{aligned} \tag{B.5}$$

where suffixes denote differentiation. The horizontal divergence of $\boldsymbol{\xi}'$, which is small for the quasi-geostrophic motion under consideration, has been neglected in the last step.

Finally, due to the assumed quasi-geostrophic scaling, the deviations of ρ and $\partial\theta/\partial z$ from their zonally symmetric basic-state values ρ_{B} and $\partial\theta_{\mathrm{B}}/\partial z$ can be neglected completely. The leading-order form of (17) can then be written as

$$\dot{M} = \int_{z_1}^{z_2}\int_0^{2\pi R} \rho_{\mathrm{B}}\left(\frac{\partial\theta_{\mathrm{B}}}{\partial z}\right)^{-1}(\mathcal{H}'+\overline{\mathcal{H}}+\boldsymbol{\xi}'\cdot\nabla\mathcal{H}')\left(\frac{\mathrm{d}R}{\mathrm{d}z}-\eta_z'\right)\mathrm{d}s\,\mathrm{d}z + O(a^3). \tag{B.6}$$

Using the definition of zonal averaging

$$2\pi R\overline{(\,)} \equiv \int_0^{2\pi R}(\,)\,\mathrm{d}s \tag{B.7}$$

in (B.6), neglecting further $O(a^3)$ terms, and noting that $\overline{(\boldsymbol{\xi}'\cdot\nabla\mathcal{H}')}$ is the Stokes correction to $\overline{\mathcal{H}}$ such that $\overline{\mathcal{H}}+\overline{(\boldsymbol{\xi}'\cdot\nabla\mathcal{H}')}=\overline{\mathcal{H}}^{\mathrm{L}}$, now yields (19).

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