Revenue optimization in posted price auctions with strategic buyers

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Posted price auctions

- Standard selling mechanism.
- Seller offers a good for a price.
- Buyer accepts or rejects.
- Repeated interactions.
Motivation

✦ Competitive advantage to the seller.
✦ Direct application to AdExchanges.
✦ Several AdExchange auctions consist of only one buyer.
✦ Develop theory for buyers and sellers interactions.
Setup

- $v \in [0, 1]$: Buyers valuation.
- $p_t$: prices offered by seller.
- $a_t = 1$ if buyer accepts the price.
- $a_t = 0$ if buyer rejects price.
- Game is played for $T$ rounds.
Strategic regret

✦ Regular regret:
\[
\max_p \sum_{t=1}^T v_{a_p,t} - \sum_{t=1}^T v_{ata_t}p_t
\]

✦ Meaningless notion for strategic buyer:

Secret value: $100.

Strategic value: $1

✦ Regret is logarithmic.
Strategic regret

✧ Regular regret:
\[
\max_p \sum_{t=1}^T v_{ap,tp} - \sum_{t=1}^T v_{tp} \]

✧ Meaningless notion for strategic buyer:

![Image showing YES! and Secret value: $100.]

✧ Regret is logarithmic.
Objectives

✦ Discounting factor $\gamma$.

✦ Buyer maximizes discounted surplus:

$$\sum_{t=1}^{T} \gamma^{t-1} a_t (v - p_t)$$

✦ Seller minimizes strategic regret.

$$Tv - \sum_{t=1}^{T} va_t p_t$$
Previous work

- **MONOTONE** algorithm [Amin et al. ’13].
- Prices: $p_t = \beta^{t-1}$, for $\beta < 1$.
- If price is accepted offer price for remaining rounds.
- Regret in $O(T\sqrt{T})$ or $O(\sqrt{T\gamma T})$ if $\gamma$ is known, for $T_\gamma = \frac{1}{1 - \gamma}$.
- Regret of any algorithm in $\Omega(T_\gamma)$. 

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Monotone algorithms

✦ General monotone algorithm.

✦ Regret $\Omega(\sqrt{TT_\gamma} + \sqrt{T})$.

✦ **MONOTONE** algorithm is optimal in its class.

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**Algorithm 1** Family of monotone algorithms.

Let $p_1 = 1$ and $p_t \leq p_{t-1}$ for $t = 2, \ldots T$.

$t \leftarrow 1$

$p \leftarrow p_t$

Offer price $p$

**while** (Buyer rejects $p$) **and** ($t < T$) **do**

$t \leftarrow t + 1$

$p \leftarrow p_t$

Offer price $p$

**end while**

**while** ($t < T$) **do**

$t \leftarrow t + 1$

Offer price $p$

**end while**
Truthful algorithm

✧ Active interval $[a, b]$ and search parameter $\epsilon$.
✧ Offer prices $a, \ldots, a + k\epsilon \ldots$ until rejected.
✧ If price $a + k\epsilon$ is rejected, update search parameter and active interval to $\epsilon^2$.
and $[a + (k - 1)\epsilon, a + k\epsilon]$. 

Modified algorithm

✦ Offer prices of truthful algorithm.

✦ If price is rejected, offer for $r$ rounds.

✦ Buyer incentivized to be truthful.

Algorithm 2 Definition of $A_r$.

\[
\begin{align*}
\text{n} &= \text{the root of } \mathcal{I}(T) \\
\text{while Offered prices less than } T \text{ do} \\
& \quad \text{Offer price } p_n \\
& \quad \text{if Accepted then} \\
& \quad \quad n = r(n) \\
& \quad \text{else} \\
& \quad \quad \text{Offer price } p_n \text{ for } r \text{ rounds} \\
& \quad \quad n = l(n) \\
& \quad \text{end if} \\
& \text{end while}
\end{align*}
\]
Regret analysis.

✧ Regret bounded by:

\[(vr + 1)(\lceil \log_2 \log_2 T \rceil + 1) + \frac{(1 + \gamma)^r TT \gamma}{2(1 - \gamma^r)}\]

✧ With knowledge of gamma:

\[(2v\gamma T \gamma \log cT + 1 + v)(\log_2 \log_2 T) + 4T \gamma.\]
Empirical results
Conclusion

✦ Regret analysis of posted price auctions with strategic buyers.
✦ (Almost) optimal algorithm.
✦ Favorable empirical results.
✦ Can we generalize this analysis to more complicated scenarios?