Problem Set #2

Exercise 1: (⋆)
Compute $\gcd(935, 1122)$ and $\gcd(1876, 4534)$ by using Euclidean algorithm.

Exercise 2: (⋆)
Find two integers $a$ and $b$ such that $a \cdot 244 + b \cdot 313 = \gcd(244, 313)$.

Exercise 3: (⋆)
Prove that if $n$ is odd, then $n^2 - 1$ is divisible by 8.

Exercise 4: (⋆)
Which of the following equations have integer solutions? (Justify your answer but do not find solutions.)
1. $51x - 7y = 88$
2. $11x - 66y = 0$
3. $33x + 44y = 1$

Exercise 5: (⋆)
Determine all the integer solutions of the equation:

$$4x + 7y = 117$$

Exercise 6: (⋆⋆) A positive integer $m$ has the prime decomposition $2^4 p_1 p_2 p_3$, where $p_1$, $p_2$, $p_3$ are some odd prime number (not necessarily distinct). The integer $m + 100$ has the prime decomposition $5 q_1 q_2 q_3$ where $q_1$, $q_2$, $q_3$ are some prime number different from 5 (not necessarily distinct). The integer $m + 200$ has the prime decomposition $23 r_1 r_2 r_3 r_4$, where $r_1$, $r_2$, $r_3$, $r_4$ are some prime number different from 23 (not necessarily distinct). Find $m$.

Exercise 7: (⋆) Is 211 prime? (Give a justification to your answer).

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1(⋆) = easy , (⋆⋆) = medium, (⋆⋆⋆) = challenge