Homework # 4

1. Let n be an integer. State and prove by the contrapositive of the following implication.
 "If \( n^2 \) is odd then n is odd"
Did you prove the implication?

2. Write down the contrapositive of the following implications
   (a) "If all men are mortal then Socrates is mortal."
   (b) "If the reals x and y are distinct then the reals (x+1)(y-1) and (x-1)(y+1) are distinct"
   (c) \( \forall x \in \mathbb{R}, [\forall \varepsilon > 0, ||f(x)|| < \varepsilon] \Rightarrow (f(x)=0) \)

3. Let \( x \in \mathbb{R} \)
   Prove by contrapositive that
   "\( x^3=2 \Rightarrow x < 2 \)"

4. Write the contrapositive of the following proposition and prove it "All integers greater than 3 and prime is odd"

5. The goal of this exercise is to prove by contrapositive the following proposition
 "If the integer \( (n^2-1) \) is divisible by 8, then the integer n is even."
   (a) Write the sentence as a mathematical formula.
   (b) Write the contrapositive in the formula you denoted in (a).
   Note that any odd integer can be written as \( m=4k+n \) with \( k \in \mathbb{N} \) and \( n \in [1,3] \)
   Justify it.
   Prove that the formula of question (b) is true.
   (d) Did we prove the first implication?