Example 1.1
Consider the following sentences: Determine if its a statement or not and its value.
(a) Two plus two equals four.
(b) Every continuous function is differentiable.
(c) $x^2 - 5x + 6 = 0$.
(d) A circle is the only convex set in the plane that has the same width in each direction.
(e) Every even number greater than 2 is the sum of two primes.

Practice 1.2
Which of the sentences are statements?
(a) If $x$ is a real number, then $x^2$.
(b) Seven is a prime number.
(c) Seven is an even number.
(d) This sentence is false.

Practice 1.3
Let $p$, $q$, $r$ be given as follows:
$p$: Today is Monday.
$q$: Five is an even number.
$r$: The set of integers is countable.
Find the negation of $p$, $q$, $r$

Practice 1.4
Compute the Truth table for $p \land q$

Practice 1.5
Identity the antecedent and the consequent in each of the following statements.
(a) If $n$ is an integer, then $2n$ is an even number.
(b) You can work here only if you have a college degree.
(c) The car will not run whenever you are out of gas.
(d) Continuity is a necessary condition for differentiability.

**Practice 1.6**
Construct a truth table for each of the following compound statements.

(a) \( \neg (p \land q) \iff [(\neg p) \lor (\neg q)] \)
(b) \( \neg (p \lor q) \iff [(\neg p) \land (\neg q)] \)
(c) \( \neg (p \Rightarrow q) \iff [p \land (\neg q)] \)

**Example 1.8**
Write the negation of *The Set S is compact and convex.*

**Practice 1.9**
Use the Tautologies in Practice 1.7 to write out a negation of each of the following statements.

(a) \( \neg (p \lor q) \iff [p \land (\neg q)] \)
(b) \( \neg (p \Rightarrow q) \iff [p \land (\neg q)] \)
(c) \( \neg (p \land q) \Rightarrow n) \iff [p \land q) \Rightarrow (\neg n)] \)

**Exercises**

1. Mark each statement True Or False. Justify each answer.
   (a) In order to be classified as a statement, a sentence must be true.
   (b) Some statements are both true and false.
   (c) When a statement \( p \) is true, its negation \( \neg p \) is false.
   (d) A statement and its negation may both be false.
   (e) In mathematical logic, the word "or" has an inclusive meaning.

2. Mark each statement True or False. Justify each answer.
   (a) In an implication \( p \rightarrow q \), statement \( p \) is referred to as the proposition.
   (b) The only case where \( p \rightarrow q \) is false is when \( p \) is true and \( q \) is false.
   (c) "If \( p \) then \( q " \) is equivalent to "\( p \) whenever \( q \)"
   (d) The negation of a conjunction is the disjunction of the negation of the individual parts.
   (e) The negation of \( p \rightarrow q \) is \( q \rightarrow p \)

3. Write the negation of each statement:
   (a) The 3x3 identity matrix is singular.
   (b) The function \( f(x) = \sin x \) is bounded on \( \mathbb{R} \).
   (c) The functions \( f \) and \( g \) are linear.
   (d) Six is prime or seven is odd
   (e) If \( x \) is in \( D \), then \( f(x) < 5 \).
   (f) If \( a \) is monotone and bounded, then \( a \) is convergent
(g) (e) If f is injective, then S is finite or denumerable.

4. Write the negation of each statement:
(a) The function \( f(x) = x^2 - 9 \) is continuous at \( x=3 \).
(b) The relation R is reflexive or symmetric
(c) Four and nine are relatively prime.
(d) \( x \) is in A or \( x \) is not in B.
(e) If \( x < 7 \), then \( f(x) \) is not in C
(f) If a is convergent, then a is monotone and bounded
(g) If f is continuous and A is open, then f inverse is open

5. Identify the antecedent and the consequent in each statement.
(a) M has a zero eigenvalue whenever M is singular.
(b) Linearity is a sufficient condition for continuity.
(c) A sequence is Cauchy only if it is bounded.
(d) \( x < 3 \) provided that \( y > 5 \).

6. Identify the antecedent and the consequent in each statement.
(a) A sequence is convergent if it is Cauchy.
(b) Convergence is a necessary condition for boundedness.
(c) Orthogonality implies invertability
(d) \( k \) is closed and bounded only if \( K \) is compact.

7. Construct a truth table for each statement.
(a) \( p \Rightarrow \neg q \)
(b) \([p \land (p \Rightarrow q)] \Rightarrow q \)
(c) \([p \Rightarrow (q \land \neg q)] \iff \neg p \)

8. Construct a truth table for each statement.
(a) \( p \lor \neg q \) (b) \( p \land \neg q \)
(c) \([\neg q] \land (p \Rightarrow q)] \Rightarrow \neg p \)

9. Indicate whether each statement is True or False.
(a) \( 3 \leq 5 \) and 11 is odd.
(b) \( 3^2 = 8 \) or \( 2 + 3 = 5 \).
(c) \( 5 > 8 \) or \( 3 \) is even.
(d) If \( 6 \) is even, then \( 9 \) is odd.
(e) If \( 8 < 3 \), then \( 2^2 = 5 \).
(f) If \( 7 \) is odd, then \( 10 \) is prime.
(g) If \( 8 \) is even and \( 5 \) is not prime.
(h) If \( 3 \) is odd or \( 4 > 6 \), then \( 9 \leq 5 \).
(i) If both \(5 - 3 = 2\) and \(5 + 3 = 2\), then \(9 = 4\)
(j) It is not the case that \(5\) is even or \(7\) is prime.

10. Indicate whether each statement is True or False.
(a) \(2 + 3 = 5\) and \(5\) is even.
(b) \(3 + 4 = 5\) or \(4 + 5 = 6\)
(c) \(7\) is even or \(6\) is not prime.
(d) If \(4 + 4 = 8\), then \(9\) is prime.
(e) If \(6\) is prime, then \(8 < 6\).
(f) If \(6 < 2\), then \(4 + 4 = 8\)
(g) If \(8\) is prime or \(7\) is odd, then \(9\) is even.
(h) If \(2 + 5 = 7\) only if \(3 + 4 = 8\), then \(3^2 = 9\).
(i) If both \(5 - 3 = 2\) and \(5 + 3 = 8\), then \(8 - 3 = 4\)
(j) It is not the case that \(5\) is not prime and \(3\) is odd.

11. Let \(p\) be the statement "The figure is a polygon" and let \(q\) be the statement "The figure is a circle." Express each of the following statements in symbols.
(a) The figure is a polygon, but it is not a circle.
(b) The figure is a polygon or a circle, but not both.
(c) If the figure is not a circle, then it is a polygon.
(d) The figure is a circle whenever it is not a polygon.

12. Let \(m\) be the statement "\(x\) is perpendicular to \(M\)." and let \(n\) be the statement "\(x\) is perpendicular to \(N\)." Express each of the following statements in symbols.
(a) \(x\) is perpendicular to \(N\) but not perpendicular to \(M\).
(b) \(x\) is not perpendicular to \(M\), nor is it perpendicular to \(N\).
(c) \(x\) is perpendicular to \(N\) only if \(x\) is perpendicular to \(M\).
(d) \(x\) is not perpendicular to \(N\) provided it is perpendicular to \(M\).
(e) It is not the case that \(x\) is perpendicular to \(M\) and perpendicular to \(N\).

13. Define a new sentential connective \(\nabla\), called nor, on the following table.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>(p \nabla q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>

(a) Use a truth table to show that \(p \nabla p\) is logically equivalent to \(\neg p\).

(b) Complete a truth table for \((p \nabla p) \nabla (q \nabla q)\).
(c) Which of our basic connectives \((p \land q)\), \((p \lor q)\), \(p \Rightarrow q\), \(p \Leftrightarrow q\), is logically equivalent
to \((p \lor p) \lor (q \lor q)\)

14. Use the truth tables to verify that each of the following is a tautology.

(a) \((p \land q) \iff (q \land p)\)

(b) \((p \lor q) \iff (q \lor p)\)

(c) \([p \land (q \land r)] \iff [(p \land q) \land r]\)

(d) \([p \lor (q \lor r)] \iff [(p \lor q) \lor r]\)

(e) \([p \land (q \lor r)] \iff [(p \land q) \lor (p \land r)]\)

(f) \(p \lor (q \land r) \iff [(p \lor q) \land (p \lor r)]\)

Construct the truth table of the following propositions \(p, q, n, s\).

(a) \((p \lor q) \text{ then } (q \lor p) \text{ then compare the two results}

(b) \([\neg (\neg p) \iff p]\)

(c) \((p \Rightarrow q) \land (q \Rightarrow p) \text{ and compare with the table of } (p \iff q)\)

(d) \(\neg (p \land q) \text{ then } (\neg p) \lor (\neg q)\)

(e) \(\neg (p \lor q) \text{ and then } (\neg p) \land (\neg q)\)

(f) \(p \lor (\neg p)\)

(g) \(\neg p \lor q \text{ and compare with } p \Rightarrow q\)

(h) \(\neg p \Rightarrow (\neg q) \text{ and compare with } p \Rightarrow q\)

(i) \([(n \Rightarrow s) \land (s \land t)] \Rightarrow (n \Rightarrow t)\)

(j) \((n \Rightarrow s) \Rightarrow [(n \land t) \Rightarrow (s \land t)]\)

\(p, q, r\) are logical propositions. Construct the truth table.

(i) \(p \Rightarrow (q \Rightarrow p)\)

(ii) \(p \Rightarrow (q \Rightarrow (p \land q))\)

Express without any \(\iff\) or \(\Rightarrow\) on the following propositions.

(i) \(\neg (p \leftrightarrow q)\)

(ii) \(\neg (p \lor q) \Rightarrow q\)

(iii) \(\neg [p \Rightarrow (q \Rightarrow r)]\)

Which of these propositions can we deduce from the following statement

*If it rains in the morning, I take my umbrella*”

\(p\): Rains in the morning \(q\): I take my umbrella

\(p \Rightarrow q\) (We are assuming this is true)
(a) "I took my umbrella so it rained this morning"
(b) "I did not take my umbrella so it was not raining this morning"
(c) "It is sunny so I did not take my umbrella"

In a newspaper the following announcement appeared: "The army will not leave the country as long as the calm is not back."

p: The army will leave the country q: Calm is back
\[ \neg q \iff \neg p \] (We are assuming this is true)

Suppose that this announcement is true, say if the following statements are true or not.
(a) "The calm is back, so the army left the country"
(b) "The army left the country so the calm is back"
(c) "The army did not leave the country so the calm is not back."

Express the following sentences with a logic connector
(a) "If the paper turns red, then the solution is acid."
(b) "The paper turns red, if the solution is acidic."

p: The paper turns red q: The solution is acidic
(c) "You will get a room only if you have no dog"

Write the negation in English and mathematically of this sentence.
(d) If there is "Cobalt", but not "Nickel" in the solution, then the paper will turn brown.

If we can replace.... by "necessary", "sufficient" "necessary and sufficient" What is the correct logic correspondent?
(a) Being 18 years old is a... condition to vote in the U.S.
(b) Getting 65 on any of the disciplines, is a condition.... to get the final exam out of high school.
(c) x=1 is a condition... for \[ x^2 = x \]
(d) x > 0 is a condition... for \[ 1 > x \]
(e) |x| = 1 is a condition ... for \[ x^2 = 1 \]