Lesson Plan

Problem Set # 8

**Exercise 1:**
Section 4.2 [F] 3., 9., 10.  

**Exercise 2:**
We say that a matrix $A \in M_n(K)$ is singular, if $Ker(L_A) \neq \{0\}$.  
1. Prove that if $A, B \in M_n(K)$ and either matrix is singular. Then $AB$ is singular.  
2. Prove that if $A$ and $B$ are both non-singular, so is $AB$.  

**Exercise 3:**
Let $T : V \rightarrow W$ be a linear operator between finite dimensional vector spaces and let $\mathcal{X} = \{e_1, ..., e_n\}$, $\mathcal{N} = \{f_1, ..., f_m\}$ be bases in $V, W$. We have defined $rank(T) = dim(R(T))$. If $A = [T]_{\mathcal{N},\mathcal{X}}$. Prove the identity $rank(T) = rank$ of the linear operator $L_A : K^n \rightarrow K^m$.  

**Exercise 4:**
Prove that the following statement are equivalent:  
1. $det(A) \neq 0$;  
2. $A$ has multiplicative inverse $A^{-1}$ such that $A^{-1}A = AA^{-1} = Id$.  
3. $L_A : K^n \rightarrow K^n$ is an invertible linear mapping (one-to-one and onto).  
Hint: You can prove $(a) \Rightarrow (b) \Rightarrow (c) \Rightarrow (a)$.  

**Exercise 5:**
If $A$ is an $n \times n$ matrix, the following conditions are equivalent:  
1. $det(A) \neq 0$ (i.e. $A$ is a nonsingular matrix and $L_A$ is invertible);  
2. the rows $R_1, ..., R_n$ are linearly independent in $K^n$;  
3. the columns $C_1, ..., C_n$ are linearly independent in $K^n$.  

1