Necessity and Luxury

$D = D(p, m)$ indicates a typical consumer's demand for a particular commodity, where p is the price and m is consumer's income.

Now consider $Y = \frac{PD}{m}$, which is the proportion of income spent on the commodity.

We would like to see what will happen if consumer's income increases:

$$\frac{\partial Y}{\partial m} = P \frac{\frac{\partial D}{\partial m} \cdot m - D}{m^2} = \frac{P}{m^2}\left(\frac{\partial D}{\partial m} m - D\right) = \frac{PD}{m^2}\left(\frac{m}{D} \cdot \frac{\partial D}{\partial m} - 1\right)$$

$$= \frac{PD}{m^2}\left(El_m D - 1\right)$$

We see that when $El_m D > 1$, $\frac{\partial Y}{\partial m} > 0$, in this case we say the commodity is a "luxury", i.e. as income increases, people tend to put more proportion of the money on it.

When $El_m D < 1$, $\frac{\partial Y}{\partial m} < 0$, in this case we say the commodity is a "necessity", i.e as income increases, people tend to put less proportion of the money on it.

Recall the example we discussed before about the demand for potatoes and apples during the Great Depression, where the income elasticity of demand for potatoes is 0.34, while that for apples is 1.32. So potatoes were necessity, while apples were luxury at that time.

Gains from Search.

Suppose you want to buy x_0 units of a commodity, and now you find it at a store, with price p_0 per unit. But you want to search among other sellers to find a lower price. Let $p(t)$ denote the lowest price per unit you expect to find after searching for t hours.

We assume $\frac{dp}{dt} < 0$, since the longer time we search, the lower price we may find.

We also assume $\frac{d^2p}{dt^2} > 0$, since it will be harder and harder to find lower price.

But searching for lower price wastes time, since otherwise you may spend your time on working, and earns hourly wage w.

The expected profit from searching for t hours is

$$\pi(t) = (p_0 - p(t))x_0 - wt$$

$\pi'(t) = -p'(t)x_0 - w$, so if t^* is a critical point, $\pi'(t^*) = -p'(t^*)x_0 - w = 0$

i.e. $p'(t^*) = -\frac{w}{x_0}$

The critical point is a maximum point, since

$\pi''(t) = -p''(t)x_0 < 0$, $\pi(t)$ is concave.

How do we interpret this result?

$p(t^*+1) - p(t^*) \approx p'(t^*)$, and $-p'(t^*)x_0 - w = 0$ implies

$$[p(t^*) - p(t^*+1)]x_0 \approx w$$

That is, in order to maximize profit, you should search until the marginal gain from searching for an extra hour is just offset by the hourly wage.

Next, we see t^* depends on x_0 and w, via $p'(t^*)x_0 + w = 0$

Let $F(x_0, w, t^*) = p'(t^*)x_0 + w$

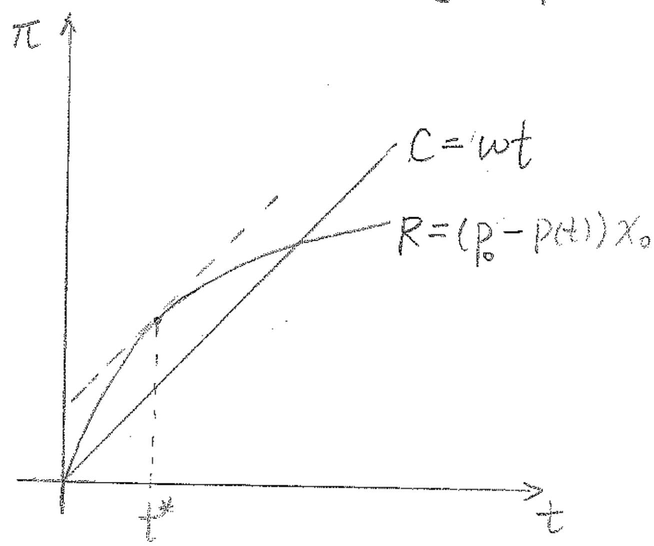
$$\frac{\partial t^*}{\partial x_0} = -\frac{\frac{\partial F}{\partial x_0}}{\frac{\partial F}{\partial t^*}} = -\frac{p'(t^*)}{p''(t^*)x_0} > 0$$

$$\frac{\partial t^*}{\partial w} = -\frac{\frac{\partial F}{\partial w}}{\frac{\partial F}{\partial t^*}} = -\frac{1}{p''(t^*)x_0} < 0$$

The implication is that: if you want to buy more units, then you're more likely to search for lower price for a longer time.

If you have high salary, then you're less likely to search for lower price for a longer time.

Instead of solving algebraically, we can also see the results by graphs:



t^* is the time at which the tangent of the curve is parallel to $C = wt$.

If x_0 increases, the curve is strached vertically, so the corresponing slope at each time becomes steeper, we need to increase the time to find a tangent line parallel to $C = wt$.

If w increases, $C = wt$ becomes steeper. Then we need to take smaller t such that the tangent line has bigger slope, in order to be parallel to $C = wt$ after w increases.

Supply and Demand Curve.

We know both the supply curve and demand curve reflect the relation between price and quantity. In fact, both supply and demand depend more factors than price. So the supply curve and demand curve we see are those whose other factors have been set constant.

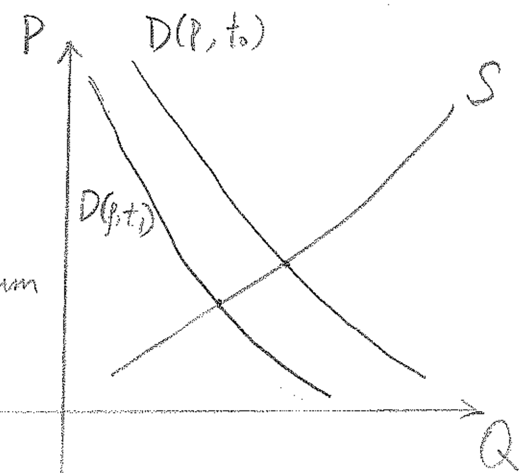
For example, the demand D is also affected by the sales tax t, as we have discussed in a previous example. So $D = D(p, t)$, and

$\frac{\partial D}{\partial p} < 0$, $\frac{\partial D}{\partial t} < 0$.

For a fixed t_0, D is a function of price p. $D = D(p, t_0)$.

There is a corresponding demand curve.

The intersection of the demand curve and the supply curve is the equilibrium, the corresponding price p^* is the equilibrium price.



Now if the tax increases from t_0 to t_1, for each price p. $D(p, t_1) < D(p, t_0)$, so the demand curve will shift to the left, then by the graph, we see the new equilibrium point goes to the left. i.e. the equilibrium price decreases, which agress with our discussion last week.