Homework V Second-Half

Due in class June 27 2017

0. Read the following Sections:
Chapter 12 Tools for Comparative Statics: 12.8 Linear Approximations, 12.9 Differentials
Chapter 13 Multivariable Optimization: 13.1 Two Variables: Necessary Conditions, 13.2 Two Variables: Sufficient Conditions, 13.3 Local Extreme Points

1. Find the linear approximation about $(0, 0)$ for the function
$$f(x, y) = e^x \ln(1 + y)$$

2. Approximate the value of $f(1.02, 1.99)$ by linear approximation, where
$$f(x, y) = 3x^2 + xy - y^2$$

3. Find the tangent plane to the surface implicitly defined by
$$x^2 + y^2 + z^2 = 3$$
at the point $(1, 1, 1)$

4. Find dU expressed in terms of dx and dy when $U = U(x, y)$ satisfies the equation
$$Ue^U = x\sqrt{y}$$

5. Solve the utility-maximizing problem $\max U = xyz$ subject to $x + 3y + 4z = 108$, by making U a function of y and z by eliminating the variable x, assuming that the critical point is a maximum point.

6. A firm produces two goods, called Alpha and Beta. The cost of producing x units of Alpha and y Units of Beta is

$$C(x, y) = x^2 + xy + y^2 + x + y + 14$$

Suppose the firm sells Alpha at price p per unit and Beta at price q per unit, and p, q are positive constants. Find the values of x and y that maximize profit.

7. Find the extreme point of the function

$$f(x, y) = -2x^2 - y^2 + 4x + 4y - 3$$

Is the extreme point a maximum point or minimum point?

8. Find all the local maximum points, local minimum points and saddle points of the function

$$f(x, y) = x^2 + 2xy^2 + 2y^2$$