Homework V Second-Half

Due in class June 27 2017

0. Read the following Sections:
   Chapter 12 Tools for Comparative Statics: 12.8 Linear Approximations, 12.9 Differentials
   Chapter 13 Multivariable Optimization: 13.1 Two Variables: Necessary Conditions, 13.2 Two Variables: Sufficient Conditions, 13.3 Local Extreme Points

1. Find the linear approximation about \((0, 0)\) for the function
   \[ f(x, y) = e^x \ln(1 + y) \]

2. Approximate the value of \(f(1.02, 1.99)\) by linear approximation, where
   \[ f(x, y) = 3x^2 + xy - y^2 \]

3. Find the tangent plane to the surface implicitly defined by
   \[ x^2 + y^2 + z^2 = 3 \]
   at the point \((1, 1, 1)\)

4. Find \(dU\) expressed in terms of \(dx\) and \(dy\) when \(U = U(x, y)\) satisfies the equation
   \[ U e^U = x \sqrt{y} \]

5. Solve the utility-maximizing problem \(\max U = xyz\) subject to \(x + 3y + 4z = 108\), by making \(U\) a function of \(y\) and \(z\) by eliminating the variable \(x\), assuming that the critical point is a maximum point.
6. A firm produces two goods, called Alpha and Beta. The cost of producing $x$ units of Alpha and $y$ units of Beta is

\[ C(x, y) = x^2 + xy + y^2 + x + y + 14 \]

Suppose the firm sells Alpha at price $p$ per unit and Beta at price $q$ per unit, and $p, q$ are positive constants. Find the values of $x$ and $y$ that maximize profit.

7. Find the extreme point of the function

\[ f(x, y) = -2x^2 - y^2 + 4x + 4y - 3 \]

Is the extreme point a maximum point or minimum point?

8. Find all the local maximum points, local minimum points and saddle points of the function

\[ f(x, y) = x^2 + 2xy + 2y^2 \]