Homework IV First-Half

Due in class June 20 2017

0. Read the following Sections:

Chapter 8 Single-Variable Optimization: Section 8.1 Introduction, 8.2 Simple Tests for Extreme Points, 8.3 Economics Examples, 8.4 The Extreme Value Theorem, 8.6 Local Extreme Points

1. The height of a flowering plant after t months is given by

$$h(t) = \sqrt{t} - \frac{1}{2}t, t \in [0, 3]$$

   At what time is the plant at its tallest?

2. Find the extreme point of the function $f(x) = x^2 + 3x - 4$. Is it a maximum point or a minimum point?

3. Find the extreme points and extreme values of the function $f(x) = x^4 - 4x^3 + 4x^2$ on the interval $[-1, 3]$

4. $f(x) = \frac{2x^2}{x^4+1}$. Show that $f'(x) = \frac{4x(1+x^2)(1+x)(1-x)}{(x^4+1)^2}$, and find the maximum value of the function on $[-2, 2]$

5. Find all the local maximum points and local minimum points of the function $f(x) = x + \frac{1}{x}$

6. Find all the local maximum points and local minimum points of $f(x) = e^{x^2} + e^{2-x^2}$

7. There is a string of length l. We want to use this string to enclose a rectangular shape region. What is the largest possible area of the region that we can enclose?

8. The tax T a person pays on gross income W is given by

$$T = a(bW + c)^p + kW$$

where a, b, c, k are positive constants and $p > 1$. Then the average tax rate is

$$\bar{T}(W) = \frac{T}{W}$$

Find the value of W that minimizes the average tax rate.