Homework V Second-Half

Due in class August 08 2017

0. Read The Following Sections:

Chapter 13 Vector Calculus: Section 13.3 The Fundamental Theorem For Line Integrals, 13.4 Green's Theorem, 13.5 Curl and Divergence

1. Determine whether this vector field is conservative:

$$\vec{F}(x,y) = < \ln y + 2xy^3, 3x^2y^2 + \frac{x}{y} >$$

2. Find a potential function of the vector field

$$\vec{F}(x,y) = < xy^2, x^2y >$$

3. Show that the line integral is independent of path and evaluate the integral:

$$\int_C 2xe^{-y}\,dx + (2y - x^2e^{-y})\,dy$$

   C is any path from $(1,0)$ to $(2,0)$.

4. Evaluate $\oint_C (y+\sqrt{x})\,dx + (2x+\cos y^2)\,dy$, where C is the boundary of the region enclosed by the parabolas along $y = x^2$ and $x = y^2$.

5. Evaluate $\oint_C y^4\,dx + 2xy^3\,dy$, where C is the ellipse $x^2 + 2y^2 = 2$

6. Use Green's Theorem to find the work done by the force field

$$\vec{F}(x,y) = < x, x^3 + 3xy^2 >$$

   in moving a particle from $(-2,0)$ to $(2,0)$ along x-axis, then moving along the semicircle $y = \sqrt{4-x^2}$ to the starting point.

7. Find the Curl and divergence of the vector field

$$\vec{F} =< \sin yz, \sin zx, \sin xy >$$

8. Prove $\mathbf{div}(\vec{F} \times \vec{G}) = \vec{G} \cdot \mathbf{curl}\vec{F} - \vec{F} \cdot \mathbf{curl}\vec{G}$

9. Is there a vector field $\vec{F}$ in $\mathbb{R}^3$ satisfying

$$\mathbf{curl}\vec{F} =< x \sin y, \cos y, z - xy >$$