Homework V First-Half

Due in class August 08 2017

0. Read The Following Sections:
   Chapter 12 Multiple Integrals: Section 12.6 Triple Integrals in Cylindrical
   Coordinates, 12.7 Triple Integrals in Spherical Coordinates
   Chapter 13 Vector Calculus: Section 13.1 Vector Fields, 13.2 Line Integral

1. Find the volume of the solid that is enclosed by the cone \( z = \sqrt{x^2 + y^2} \)
   and the sphere \( x^2 + y^2 + z^2 = 1 \) first in cylindrical coordinates, then in
   spherical coordinates.

2. Evaluate \( \iiint_E (x + y + z) \, dV \), where \( E \) is the solid in the first oc-
   tant (points with all three coordinates positive) that lies under the
   paraboloid \( z = 4 - x^2 - y^2 \).

3. Consider the circle \((x - b)^2 + z^2 = a^2 \) \((0 < a < b, \) constants). If we
   rotate this disk with respect to the \( z \)-axis, its trace forms a surface
   called torus. Compute the volume of the region enclosed by this torus.
   [After doing this exercise, you are able to compute the volume of a
   bagel when you have one for breakfast!]

4. Evaluate the integral \( \int_C xy^4 \, ds \), where \( C \) is the right half of the circle
   \( x^2 + y^2 = 16 \).

5. Evaluate the integral \( \int_C (x^2 + y^2 + z^2) \, ds \), where \( C \) is given by \( x = t, y =
   t^2, z = t^3, 0 \leq t \leq 1 \).

6. Evaluate \( \int_C z^2 \, dx + x^2 \, dy + y^2 \, dz \), where \( C \) is the line segment from
   \((1, 0, 0) \) to \((4, 1, 2) \).

7. Evaluate \( \int_C (y + z) \, dx + (x + z) \, dy + (x + y) \, dz \), where \( C \) consists of line
   segments from \((0, 0, 0) \) to \((1, 0, 1) \) and from \((1, 0, 1) \) to \((0, 1, 2) \).
8. Find the work done by the force field $\vec{F}(x, y) = \langle x, y + 2 \rangle$ in moving an object along an arch of the cycloid $\vec{r} = \langle t - \sin t, 1 - \cos t \rangle$, $0 \leq t \leq 2\pi$

9. If $C$ is a smooth curve in $\mathbb{R}^2$ given by a vector function $\vec{r}(t)$, $a \leq t \leq b$, and $\vec{v}$ is a constant vector, show that

$$\int_C \vec{v} \cdot d\vec{r} = \vec{v} \cdot [\vec{r}(b) - \vec{r}(a)]$$