Homework V First-Half

Due in class August 08 2017

0. Read The Following Sections:

Chapter 12 Multiple Integrals: Section 12.6 Triple Integrals in Cylindrical Coordinates, 12.7 Triple Integrals in Spherical Coordinates

Chapter 13 Vector Calculus: Section 13.1 Vector Fields, 13.2 Line Integral

1. Find the volume of the solid that is enclosed by the cone $z = \sqrt{x^2 + y^2}$ and the sphere $x^2 + y^2 + z^2 = 1$ first in cylindrical coordinates, then in spherical coordinates

2. Evaluate $\iiint_E (x + y + z)\, dV$, where E is the solid in the first octant (points with all three coordinates positive) that lies under the paraboloid $z = 4 - x^2 - y^2$

3. Consider the circle $(x - b)^2 + z^2 = a^2$ ($0 < a < b$, constants). If we rotate this disk with respect to the z-axis, its trace forms a surface called torus. Compute the volume of the region enclosed by this torus. [After doing this exercise, you are able to compute the volume of a bagel when you have one for breakfast!]

4. Evaluate the integral $\int_C xy^4\, ds$, where C is the right half of the circle $x^2 + y^2 = 16$

5. Evaluate the integral $\int_C (x^2 + y^2 + z^2)\, ds$, where C is given by $x = t, y = t^2, z = t^3, 0 \le t \le 1$

6. Evaluate $\int_C z^2\, dx + x^2\, dy + y^2\, dz$, where C is the line segment from $(1, 0, 0)$ to $(4, 1, 2)$

7. Evaluate $\int_C (y + z)\, dx + (x + z)\, dy + (x + y)\, dz$, where C consists of line segments from $(0, 0, 0)$ to $(1, 0, 1)$ and from $(1, 0, 1)$ to $(0, 1, 2)$.

8. Find the work done by the force field $\vec{F}(x, y) =< x, y+2 >$ in moving an object along an arch of the cycloid $\vec{r} =< t - \sin t, 1 - \cos t >, 0 \le t \le 2\pi$

9. If C is a smooth curve in $\mathbb{R}^2$ given by a vector function $\vec{r}(t), a \le t \le b$, and $\vec{v}$ is a constant vector, show that

$$\int_C \vec{v} \cdot d\vec{r} = \vec{v} \cdot [\vec{r}(b) - \vec{r}(a)]$$