# Homework III

Due in class July 25 2017

0. Read The Following Sections:

Chapter 11 Partial Derivatives: Section 11.7 Maximum and Minimum Values, 11.8 Lagrange Multiplier

1. Find all the local maximum, local minimum and saddle points of the function
$$f(x,y) = y^3 + 3x^2y - 6x^2 - 6y^2 + 2$$

2. Find the maximum and minimum of $f(x,y) = x^2 + y^2 - 2x$ on the triangular region with vertices $(2,0),(0,2),(0,-2)$

3. The Ninja Turtles are running a cafe selling pizza and bread. One day they are having a meeting.

   *Michelangelo*: The price for a pizza is 12 dollars.

   *Raphael*: The price for bread is 6 dollars.

   *Donatello*: The daily cost of producing $x$ pizza and $y$ bread is $C(x,y) = x^2 - 2xy + 2y^2 - 20x - 10y + 514$ dollars.

   *Leonardo*: It seems our products are popular, that is, every day we can sell all the pizza and bread that we have produced regardless of the production.

   Based on their conversation, can you find the daily production level $x$ and $y$ that maximize profit?

4. Find the dimensions of a rectangular box of maximum volume such that the sum of the lengths of its 12 edges is a constant $c$.

5. Three alleles (alternative versions of a gene) A,B,O determine the four blood types A(AA or AO), B(BB or BO), O(OO) and AB. The Hardy-Weinberg Law states that the proportion of individuals in a population who carry two different alleles is

$$P = 2pq + 2pr + 2rq$$

where $p, q, r$ are the proportions of $A, B, O$ in the population. Use the fact $p + q + r = 1$ to show that $P$ is at most $\frac{2}{3}$

6. If a triangle has three edges of length $a, b, c$, by Heron's Formula, its area is given by

$$\sqrt{s(s-a)(s-b)(s-c)}$$

where $s = \frac{a+b+c}{2}$ is half of the perimeter.

If it is known the perimeter of a triangle is $2s$, what is the largest possible area of the triangle?

7. Find the largest and smallest volumes of a rectangular box whose surface area is 1500 and whose total edge length is 200

8. The plane $x + y + 2z = 2$ intersects the paraboloid $z = x^2 + y^2$ in an ellipse. Find the points on this ellipse that are nearest to and farthest from the origin.

9. A triangle has all its three vertices on a circle of radius $r$. Find the largest possible area of the triangle.

Hint: You may use the Sine Theorem, which states that for a triangle $\triangle ABC$ with a circumcircle of radius $r$, the following equalities hold:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2r$$