Homework II First-Half

Due in class July 18 2017

0. Read The Following Sections:

Chapter 10 Vectors and the Geometry of Space: Section 10.7 Vector Functions and Spae Curves, 10.8 Arc Length and Curvature (the part for Curvature is recommended, although not required)

Chapter 11 Partial Derivatives: Section 11.1 Functions of Several Variables, Section 11.2 Limits and Continuity

1. Find the tangent vector and unit tangent vector of the space curve

$$\vec{r}(t) =< \cos t, 3t, 2\sin 2t >$$

at $t = 0$.

2. Find the point on the curve $\vec{r}(t) =< 2\cos t, 2\sin t, e^t >$, $0 \leq t \leq \pi$, where the tangent line is parallel to the plane $\sqrt{3}x + y = 1$.

3. Prove the rule $(\vec{u}(t) \times \vec{v}(t))' = \overrightarrow{u}'(t) \times \vec{v}(t) + \vec{u}(t) \times \overrightarrow{v}'(t)$.

4. Compute the length of the curve $\vec{r}(t) =< t, 3\cos t, 3\sin t >$ between $(0, 3, 0)$ and $(\pi, -3, 0)$.

5. If a curve has the property that the position vector $\vec{r}(t)$ is always perpendicular to the tangent vector $\overrightarrow{r}'(t)$, show that the curve lies on a sphere with center the origin.

6. Find the domain of $f(x, y) = \frac{|x|e^x}{\sqrt{4-x^2-y^2}} \ln(1-y)$, and draw the domain in the xy-plane.

7. Sketch the level curves of $f(x, y) = x^2 + y^2 = c$ for $c = 1, 4, 9$.

8. Does the following limit exist?

$$\lim_{(x,y)\to(0,0)} \frac{x^4 - 4y^2}{x^2 + 2y^2}$$

9. Prove that if $a \neq 1$, the following limit doesn't exist

$$\lim_{(x,y)\to(0,0)} \frac{x^a + y}{x + y}$$

10. Compute the limit:

$$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2 + y^2}}$$