1. Differentiate the functions:

   (i). $f(x) = (4x - x^2)^{100}$

   Solution: $f'(x) = 100(4x - x^2)^{99}(4 - 2x)$

   (ii). $f(x) = \frac{1}{x^2+1}$

   Solution: $f'(x) = -\frac{2x}{(x^2+1)^2}$

   (iii). $f(x) = \cos(x^3 + a^3)$, where a is a constant

   Solution: $f'(x) = -3x^2 \sin(x^3 + a^3)$

   (iv). $f(x) = (x^2 + 1)^3(x^2 + 2)^4$

   Solution:

$$\begin{aligned} f'(x) &= 3(x^2+1)^2 2x(x^2+2)^4 + 4(x^2+2)^3(x^2+1)^3 2x \\ &= 2x(x^2+1)^2(x^2+2)^3(7x^2+10) \end{aligned}$$

   (v). $f(x) = \sin(x \cos x)$

   Solution: $f'(x) = \cos(x \cos x)(\cos x - x \sin x)$

   (vi). $f(x) = \sin(\sin(\sin x))$

   Solution: $f'(x) = \cos(\sin(\sin x)) \cos(\sin x) \cos x$

2. Find $\frac{dy}{dx}$ by implicit differentiation: $y \cos x = x^2 + y^2$

   Solution: Differentiate both sides with respect to x:

   $y' \cos x - y \sin x = 2x + 2yy'$, so we get $y' = \frac{2x + y \sin x}{\cos x - 2y}$

3. Use implicit differentiation to find an equation of the tangent line to the curve $x^2 + 2xy - y^2 + x = 2$ at $(1, 2)$.

   Solution: Differentiate both sides with respect to x:

$2x + 2y + 2xy' - 2yy' + 1 = 0$, so $y' = \frac{2x+2y+1}{2y-2x}$. When $(x, y) = (1, 2)$, $y' = \frac{7}{2}$

So the tangent line is $y - 2 = \frac{7}{2}(x - 1)$

4. Find y'' of $9x^2 + y^2 = 9$ by implicit differentiation.

   Solution: Differentiate both sides of the equation, we get

   $18x + 2yy' = 0$, so $y' = -\frac{9x}{y}$

   $y'' = -\frac{9y-9xy'}{y^2} = \frac{9xy'-9y}{y^2} = -\frac{81x^2+9y^2}{y^3}$

5. Find the linearization $L(x)$ of $f(x) = \sin x$ at $x = \frac{\pi}{6}$

   Solution: $f'(x) = \cos x$, so $f'(\frac{\pi}{6}) = \frac{\sqrt{3}}{2}$.

   The linearization is $L(x) = f(\frac{\pi}{6}) + f'(\frac{\pi}{6})(x - \frac{\pi}{6}) = \frac{1}{2} + \frac{\sqrt{3}}{2}(x - \frac{\pi}{6})$

6. Find the differential of $y = x \cos x$

   Solution: $y' = \cos x - x \sin x$, so the differential is $dy = (\cos x - x \sin x)dx$

7. The radius of a circular disk is given as 24 cm with a maximum error in measurement of 0.2 cm. Use differentials to estimate the maximum error in the calculated area of the disk.

   Solution: We know the area of a disk is $A = \pi r^2$.

   $\Delta A \approx dA = A'dr = 2\pi r dr = 2\pi \times 24 \times 0.2 = 9.6\pi \approx 30.144\ cm^2$