1. Compute the derivative of $f(x) = \frac{2x+1}{x+3}$ using the definition of derivative.

   Solution:

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{\frac{2(x+h)+1}{(x+h)+3} - \frac{2x+1}{x+3}}{h} \\
&= \lim_{h\to 0} \frac{\frac{(2x+2h+1)(x+3)-(x+h+3)(2x+1)}{(x+h+3)(x+3)}}{h} \\
&= \lim_{h\to 0} \frac{5h}{(x+h+3)(x+3)h} \\
&= \lim_{h\to 0} \frac{5}{(x+h+3)(x+3)} \\
&= \frac{5}{(x+3)^2}
\end{aligned}
$$

2. Find an equation of the tangent line to the graph of $f(x) = \sqrt{x}$ at $(1, 1)$.

   Solution:

$$
\begin{aligned}
f'(1) &= \lim_{h\to 0} \frac{\sqrt{1+h} - \sqrt{1}}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{1+h} - 1}{h} \frac{\sqrt{1+h} + 1}{\sqrt{1+h} + 1} \\
&= \lim_{h\to 0} \frac{(1+h) - 1}{h(\sqrt{1+h} + 1)} \\
&= \lim_{h\to 0} \frac{h}{h(\sqrt{1+h} + 1)} \\
&= \lim_{h\to 0} \frac{1}{\sqrt{1+h} + 1} \\
&= \frac{1}{2}
\end{aligned}
$$

So the equation of the tangent line is

$$y - 1 = \frac{1}{2}(x - 1)$$

3. If a ball is thrown into the air with a velocity of 40 m/s, its height (in meters) after t seconds is given by $y = 40t - 5t^2$. Find the velocity if the ball after 2 seconds. Is it going upward or downward at that time?

   Solution:

$$\begin{aligned} v(2) &= \lim_{h\to 0} \frac{[40(2+h) - 5(2+h)^2] - [40 \times 2 - 5 \times 2^2]}{h} \\ &= \lim_{h\to 0} \frac{20h - 5h^2}{h} \\ &= \lim_{h\to 0} 20 - 5h \\ &= 20 \end{aligned}$$

   So the velocity after 2 seconds is 20, going upward since it is positive.

4. $f(x) = \sqrt[3]{x}$. Show that $f'(0)$ does not exist. [Hint: $(a-b)(a^2+ab+b^2) = a^3 - b^3$]

   Solution:

$$\begin{aligned} f'(0) &= \lim_{h\to 0} \frac{\sqrt[3]{0+h} - \sqrt[3]{0}}{h} \\ &= \lim_{h\to 0} \frac{h^{\frac{1}{3}}}{h} \\ &= \lim_{h\to 0} \frac{1}{h^{\frac{2}{3}}} \\ &= \lim_{h\to 0} \frac{1}{\sqrt[3]{x^2}} \\ &= \infty \end{aligned}$$

   So $f'(0)$ does not exist.

5. On which intervals is $f(x) = |x-6|$ differentiable?

   Solution:

   If $x > 6$:

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{|x+h-6| - |x-6|}{h} \\
&= \lim_{h\to 0} \frac{(x+h-6) - (x-6)}{h} \\
&= \lim_{h\to 0} \frac{h}{h} \\
&= \lim_{h\to 0} 1 \\
&= 1
\end{aligned}
$$

   If $x < 6$:

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{|x+h-6| - |x-6|}{h} \\
&= \lim_{h\to 0} \frac{(6-(x+h)) - (6-x)}{h} \\
&= \lim_{h\to 0} \frac{-h}{h} \\
&= \lim_{h\to 0} -1 \\
&= -1
\end{aligned}
$$

   if $x = 6$: we are going to show $f'(6)$ doesn't exist since

$$
\lim_{h\to 0^-} \frac{|x+h-6| - |x-6|}{h} \neq \lim_{h\to 0^+} \frac{|x+h-6| - |x-6|}{h}
$$

$$
\begin{aligned}
\lim_{h\to 0^-} \frac{|x+h-6|-|x-6|}{h} &= \lim_{h\to 0^-} \frac{(6-(x+h))-(6-x)}{h} \\
&= \lim_{h\to 0^-} \frac{-h}{h} \\
&= \lim_{h\to 0^-} -1 \\
&= -1
\end{aligned}
$$

while

$$
\begin{aligned}
\lim_{h\to 0^+} \frac{|x+h-6|-|x-6|}{h} &= \lim_{h\to 0^+} \frac{|x+h-6|-|x-6|}{h} \\
&= \lim_{h\to 0^+} \frac{(x+h-6)-(x-6)}{h} \\
&= \lim_{h\to 0^+} \frac{h}{h} \\
&= \lim_{h\to 0^+} 1 \\
&= 1
\end{aligned}
$$

6. $f(x) = 3x^2$. Compute f', f'', f''' using definition of derivative.

   Solution:

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\
&= \lim_{h\to 0} \frac{3(x+h)^2-3x^2}{h} \\
&= \lim_{h\to 0} \frac{6xh+3h^2}{h} \\
&= \lim_{h\to 0} 6x+3h \\
&= 6x
\end{aligned}
$$

$$\begin{aligned}
f''(x) &= \lim_{h\to 0} \frac{f'(x+h)-f'(x)}{h} \\
&= \lim_{h\to 0} \frac{6(x+h)-6x}{h} \\
&= \lim_{h\to 0} \frac{6h}{h} \\
&= \lim_{h\to 0} 6 \\
&= 6
\end{aligned}$$

$$\begin{aligned}
f'''(x) &= \lim_{h\to 0} \frac{f''(x+h)-f''(x)}{h} \\
&= \lim_{h\to 0} \frac{6-6}{h} \\
&= \lim_{h\to 0} 0 \\
&= 0
\end{aligned}$$

7. Prove the derivative of an even function is an odd function.

   Solution: If f is an even function,

$$\begin{aligned}
f'(-x) &= \lim_{h\to 0} \frac{f(-x+h)-f(-x)}{h} \\
&= \lim_{h\to 0} \frac{f(x-h)-f(x)}{h} \\
&= \lim_{k\to 0} \frac{f(x+k)-f(x)}{-k} \\
&= -\lim_{k\to 0} \frac{f(x+k)-f(x)}{k} \\
&= -f'(x)
\end{aligned}$$

   So f' is an odd function.