1. Find the domain of the following functions:
   (i). \( f(x) = \frac{1}{\sqrt{x^2 - 5x + 4}} \)
   \[ \text{Solution: We need } x^2 - 5x + 4 > 0, \text{ i.e. } (x - 1)(x - 4) > 0, \text{ so } x < 1 \text{ or } x > 4. \]
   The domain is \((-\infty, 1) \cup (4, +\infty)\)

   (ii). \( f(x) = \frac{\sqrt{x+1}}{x-2} \)
   \[ \text{Solution: We need } x + 1 \geq 0 \text{ and } x - 2 \neq 0. \] 
   So \( x \geq -1 \) and \( x \neq 2 \), the domain is \([-1, 2) \cup (2, +\infty)\)

   (iii). \( f(x) = \frac{x^2}{x} \)
   \[ \text{Solution: We need } x \neq 0, \text{ so the domain is } (-\infty, 0) \cup (0, +\infty) \]

2. Determine whether the following functions are odd or even:
   (i). \( f(x) = \frac{x^2+1}{x^4-2} \)
   \[ \text{Solution: } f(-x) = \frac{(-x)^2+1}{(-x)^4-2} = \frac{x^2+1}{x^4-2} = f(x), \text{ so } f \text{ is an even function.} \]

   (ii). \( f(x) = x|x| \)
   \[ \text{Solution: } f(-x) = (-x)|-x| = -x|x| = -f(x), \text{ so } f \text{ is an odd function.} \]

3. Determine the domain of \( f \circ g \) in each case:
   (i). \( f(x) = \frac{1}{x^2}, \ g(x) = \cos x \)
   \[ \text{Solution: } f \circ g(x) = f(g(x)) = \frac{1}{g(x)^2} = \frac{1}{\cos^2 x}, \text{ so we need } \cos x \neq 0, \text{ which means } x \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \]
   \[ \text{So the domain is } \{x \in \mathbb{R} | x \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z}\} \]

   (ii). \( f(x) = \sqrt{x-1}, \ g(x) = x^2 \)
   \[ \text{Solution: } f \circ g(x) = f(g(x)) = \sqrt{g(x)^2 - 1} = \sqrt{x^2 - 1}, \text{ so we need } x^2 - 1 \geq 0, \]
   \[ \text{i.e. } x \leq -1 \text{ or } x \geq 1. \text{ So the domain is } (-\infty, -1] \cup [1, +\infty). \]
4. Describe how to obtain the graph of \( f(x) = x^2 - 4x + 3 \) from that of \( g(x) = x^2 \).

**Solution:** \( f(x) = x^2 - 4x + 3 = (x - 2)^2 - 1 \), so we can shift the graph of \( g(x) = x^2 \) to the right by 2 units, then shift downward by 1 unit to get the graph of \( f(x) \).

5. \( f(x) = mx + b \) is a linear function. \( f(0) = 3 \) and \( f(1) = 2 \). Find the expression of \( f(x) \).

**Solution:** Since \( f(0) = 3 \) and \( f(1) = 2 \), we get the two equations:

\[
\begin{align*}
  f(0) &= b = 3 \\
  f(1) &= m + b = 2 
\end{align*}
\]

So we get \( b = 3 \) and \( m = -1 \), \( f(x) = -x + 3 \).

6. \( f(x) \) is a quadratic function. Its graph has vertex at \((1, 2)\) and the y-intercept is \( y = -2 \). Find the expression of \( f(x) \).

**Solution:** Assume \( f(x) = ax^2 + bx + c \). It has y-intercept \( y = -2 \), so \( c = -2 \), \( f(x) = ax^2 + bx - 2 \). Since its vertex is at \((1, 2)\), the symmetric axis is \( x = \frac{-b}{2a} = 1 \), so \( b = -2a \). And \( f(1) = a + b - 2 = 2 \), so \( a + b = 4 \). Solving

\[
\begin{align*}
  b &= -2a \\
  a + b &= 4 
\end{align*}
\]

We get \( a = -4 \), \( b = 8 \), so \( f(x) = -4x^2 + 8x - 2 \)