1. Find the domain of the following functions:

   (i). $f(x) = \frac{1}{\sqrt{x^2-5x+4}}$

   Solution: We need $x^2 - 5x + 4 > 0$, i.e. $(x-1)(x-4) > 0$, so $x < 1$ or $x > 4$. The domain is $(-\infty, 1) \cup (4, +\infty)$

   (ii). $f(x) = \frac{\sqrt{x+1}}{x-2}$

   Solution: We need $x + 1 \geq 0$ and $x - 2 \neq 0$. So $x \geq -1$ and $x \neq 2$, the domain is $[-1, , 2) \cup (2, +\infty)$

   (iii). $f(x) = \frac{x^2}{x}$

   Solution: We need $x \neq 0$, so the domain is $(-\infty, 0) \cup (0, +\infty)$

2. Determine whether the following functions are odd or even:

   (i). $f(x) = \frac{x^2+1}{x^4-2}$

   Solution: $f(-x) = \frac{(-x)^2+1}{(-x)^4-2} = \frac{x^2+1}{x^4-2} = f(x)$, so f is an even function.

   (ii). $f(x) = x|x|$

   Solution: $f(-x) = (-x)|-x| = -x|x| = -f(x)$, so f is an odd function.

3. Determine the domain of $f \circ g$ in each case:

   (i). $f(x) = \frac{1}{x^2}$, $g(x) = \cos x$

   Solution: $f \circ g(x) = f(g(x)) = \frac{1}{g(x)^2} = \frac{1}{\cos^2 x}$, so we need $\cos x \neq 0$, which means $x \neq \frac{\pi}{2} + k\pi$ for any integer k. So the domain is $\{x \in \mathbb{R} | x \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z}\}$

   (ii). $f(x) = \sqrt{x-1}$, $g(x) = x^2$

   Solution: $f \circ g(x) = f(g(x)) = \sqrt{g(x)^2 - 1} = \sqrt{x^2 - 1}$, so we need $x^2 - 1 \geq 0$, i.e. $x \leq -1$ or $x \geq 1$. So the domain is $(-\infty, -1] \cup [1, +\infty)$.

4. Describe how to obtain the graph of $f(x) = x^2 - 4x + 3$ from that of $g(x) = x^2$.

   Solution: $f(x) = x^2 - 4x + 3 = (x-2)^2 - 1$, so we can shift the graph of $g(x) = x^2$ to the right by 2 units, then shift downward by 1 unit to get the graph of $f(x)$.

5. $f(x) = mx + b$ is a linear function. $f(0) = 3$ and $f(1) = 2$. Find the expression of $f(x)$.

   Solution: Since $f(0) = 3$ and $f(1) = 2$, we get the two equations:

$$\begin{cases} f(0) = b = 3 \\ f(1) = m + b = 2 \end{cases}$$

   So we get $b = 3$ and $m = -1$, $f(x) = -x + 3$.

6. $f(x)$ is a quadratic function. Its graph has vertex at $(1, 2)$ and the y-intercept is $y = -2$. Find the expression of $f(x)$.

   Solution: Assume $f(x) = ax^2 + bx + c$. It has y-intercept $y = -2$, so $c = -2$, $f(x) = ax^2 + bx - 2$. Since its vertex is at $(1, 2)$, the symmetric axis is $x = -\frac{b}{2a} = 1$, so $b = -2a$. And $f(1) = a + b - 2 = 2$, so $a + b = 4$. Solving

$$\begin{cases} b = -2a \\ a + b = 4 \end{cases}$$

   We get $a = -4$, $b = 8$, so $f(x) = -4x^2 + 8x - 2$