1. Find all the elements in \( \mathbb{Z}/12\mathbb{Z} \) that have multiplicative inverse.

2. What’s the order of \( \text{Aut}(\mathbb{Z}/8\mathbb{Z}) \)? What’s the order of \( \text{Aut}(\mathbb{Z}/10\mathbb{Z}) \)? Are they isomorphic?

3. The set of nonzero complex numbers \( \mathbb{C}^\times \) is a group with multiplication of complex numbers as law of composition. The norm of a complex number \( z = x + iy \), where \( x, y \in \mathbb{R} \), is \( |z| = \sqrt{x^2 + y^2} \). It is known that given two complex numbers \( z_1, z_2 \in \mathbb{C} \), \( |z_1z_2| = |z_1||z_2| \). The unit circle is defined to be \( S^1 = \{ z \in \mathbb{C} : |z| = 1 \} \).
   (i). Show that \( S^1 \) is a normal subgroup of \( \mathbb{C}^\times \).
   (ii). If \( \mathbb{R}^{>0} \) is the group of all positive real numbers with multiplication of real numbers as law of composition, prove \( \mathbb{C}/S^1 \cong \mathbb{R}^{>0} \).

4. \( f : G \rightarrow G' \) is a group homomorphism, and \( H \) is a normal subgroup of \( G \) such that \( H \subseteq \ker(f) \). \( \pi : G \rightarrow G/H \) is the quotient map. Prove there is a unique homomorphism \( F : G/H \rightarrow G' \) such that \( f = F \circ \pi \), and \( F \) is injective if and only if \( H = \ker(f) \).

5. \( G \) is a group. \( H \) is a subgroup of \( G \) and \( N \) is a normal subgroup of \( G \).
   \( HN = \{ hn \in G | h \in H, n \in N \} \).
   (i). Prove that \( HN \) is a subgroup of \( G \).
   (ii). Prove that \( H/(H \cap N) \cong HN/N \). (Hint: consider \( f : H \rightarrow HN/N \) given by \( f(h) = hN \))

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