Grading Criteria for the Midterm

Problem 1. (10 pts) Apply chain rule to calculate partial derivatives — 2-3 pts each; put the terms together — 2 pts.

Problem 2. (30 pts)

(a) (10 pts) Find out equations of characteristics — 7 pts; write down solution — 3 pts.

(b) (5 pts) The direction of axes is not important (); whether or not the characteristics for negative \( t \) are drawn is not important either;

(c) (15 pts) Write down the ODE for \( u \) along the characteristics — 6 pts; write down the integral form of \( u \) along the characteristics with correct initial data — 6 pts; get the correct form of solution in terms of \( x \) and \( t \) — 3 pts.

Apply Duhamel’s principle in a wrong way — at most 10 pts out of 15.

The final solution should contain an integral of \( f \) (say in \( \tau \)), and both arguments of \( f \) should depend on \( \tau \); otherwise — at most 10 pts out of 15.

Problem 3. (40 pts) Classify the equation — 10 pts; find out the change of variable — 15 pts; get the general solution — 5 pts; do some algebra to find out the solution for the initial value problem — 10 pts.

Various mistakes could have been made in finding out the correct change of variable. Here are some rules: wrong factorization of the differential operator — at most 11 pts out of 15 (depending on how wrong it could be); if some coefficients have wrong signs or some coefficients are mistakenly swapped — at most 12 pts out of 15 (depending on how much detail you have showed).

There are several steps in doing the algebra to get the final solution: plug in initial conditions to get two equations of arbitrary functions \( F \) and \( G \) — 2 pts; differentiate one of the two equations — 2 pts; solve for \( F' \) and \( G' \) — 2 pts; if you alternatively took integral instead of taking differentiation, taking integral is worth 4 pts; solve for \( F \) and \( G \) — 2 pts; write down the final solution — 2 pts.

If mistakes have been made before the final algebra, which you managed to do correctly based on what you had got, partial points will be given for the efforts.

Problem 4. (20 pts)

(a) (7.5 pts) No comments.

(b) (5 pts) No comments.

(c) (7.5 pts) You would get at least 4 pts if you mentioned the initial data is a (2D) delta function.

(d) (10 extra pts) Definition for \( G_d \) (with or without justification) — 4 pts; show it is radial — 3 pts; initial data is delta function — 2 pts; explain why it is Green’s function — 1 pt.

Additional (a few) points will be given for useful calculation/explanation that shows partial understanding of the problems. Additional points will be taken for mistakes in calculation such as missing the constant (initial data) in integrals.