Meromorphic  A function $f$ is meromorphic in a domain $D$ if it is analytic in $D$ except for poles.

Winding Number  Let $\Gamma$ be a simple closed contour that does not pass through $0$. The winding number of $\Gamma$ is the number of times the contour winds around the origin, which is positive if it goes around counterclockwise, and negative if it goes around clockwise. If $C$ is a positively oriented simple closed contour, $f$ is meromorphic in the domain enclosed by $C$ and nonzero on $C$, and $\Gamma = f(C)$, then the winding number of $\Gamma$ can be calculated from the change in the argument of $f(z)$ as $f$ goes around $C$:

\[
\text{winding number of } \Gamma = \frac{1}{2\pi} \Delta_C \arg f(z).
\]

Argument Principle  Suppose $f$ is meromorphic in the domain interior to a positively oriented simple closed contour $C$, and $f$ is analytic and non-zero on $C$. Let $Z$ be the number of zeros of $f$ inside $C$, including multiplicity, and $P$ be the number of poles. Then

\[
\frac{1}{2\pi i} \int_C \frac{f'(z)}{f(z)} \, dz = \frac{1}{2\pi} \Delta_C \arg f(z) = Z - P.
\]

Rouche’s Theorem  Suppose that $f$ and $g$ are analytic inside and on a simple closed contour $C$, and $|f(z)| > |g(z)|$ at each point on $C$. Then $f$ and $f + g$ have the same number of zeros, counting multiplicities, inside $C$. 

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