Continuous Time Finance, Spring 2018
NYU Courant Institute
Prof. Bruno Dupire, and Monty Essid
Wednesdays 7:10-9:00pm, WWH 1302

Prerequisites: Derivative Securities and Stochastic Calculus, or equivalent

Course Description: This is a second course in arbitrage-based pricing of derivative securities. Concerning equity and FX models: We discuss numerous approaches that are used in practice in these markets, such as the local volatility model, Heston, SABR, and stochastic local volatility. The discussion will include calibration and hedging issues and the pricing of the most common structured products.

Concerning interest rate models: We start with a thorough discussion of one-factor short-rate models (Vasicek, CIR, Hull-White) then proceed to more advanced topics such as two-factor Hull-White, forward rate models (HJM) and the LIBOR market model. Throughout, the pricing of specific payoffs will be considered and practical examples and insights will be provided. We give an introduction to inflation models.

We cover a few special topics: We provide an introduction to stochastic optimal control with applications, as well as optimal stopping time theory and its application to American options pricing. We introduce Cox default processes and discuss their applications to unilateral and bilateral CVA/DVA.

Website: Assignments and slides will be posted on the course website in NYU classes.

Contact Information :

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Teaching assistant/grader: Qian Shen: qian.shen.math@nyu.edu

Office hours: TBA

Textbooks: There is no required textbook for the class, but you are more than encouraged to have a look at the following references:

For the class:

- Arbitrage Theory in Continuous Time, Tomas Bjork, Oxford University press
- Stochastic Calculus for Finance II: Continuous-Time Models, Steven Shreve, Springer
- Volatility Surface: A Practitioner's Guide, Jim Gatheral, Wiley Finance

Review of Probability and Stochastic Calculus:

- Probability Essentials, Jean Jacod and Philip Protter, Springer
- Brownian Motion and Stochastic Calculus, Ioannis Karatzas and Steven Shreve, Springer

- Paul Bourgade's Lecture notes available at : `cims.nyu.edu/~bourgade/SA2010/StochasticAnalysis.pdf`

Review of Derivative Securities:

- Options, Futures, and Other Derivatives, John C. Hull, Pearson
- Bob Kohn's lecture notes available at: `math.nyu.edu/faculty/kohn/derivative_securities.html`

Coding requirements: Some assignments will include coding. Please use Python as a programming language. It is recommended to use it with the SciPy libraries, and with the Jupyter interface for clarity of the presentation.

- Coding prerequisites: very basic familiarity with a programming language. Some basic training exercises are available at `https://www.hackerrank.com/domains/python/py-introduction` and `https://www.hackerrank.com/domains/python/numpy/1`
- To install Jupyter, Python and all the recommended packages, use the Anaconda distribution `https://www.continuum.io/downloads` (Python 3.x recommended).
- Please comment your code in your assignments: you do not need to comment every line, but please include a comment explaining the purpose of every major step and of your functions.
- Make sure to run all your code, check that all outputs are correct and that there is no debugging error before turning in your assignment.

Course Requirements: There will be homework assignments on most weeks, an in-class midterm exam on Wed 3/21 , and a final exam on Wed 5/9 (in the normal class time and place). The semester grade will be based on the HW (1/3), the midterm (1/3), and the final exam (1/3).

Collaboration on homework is encouraged (homeworks are not exams) but registered students must write up and turn in their solutions individually.
If you work with other students, please include their names on your solution sheet. HW may not be turned in late; **no extension is allowed**.
Doing the HW is important, not only because it counts as part of the grade, but also because if you don't do the HW, you probably won't do well on the exams.
Copying solutions from other students (or past solution sheets) is not permitted, and not a good idea: you won't have the intended experience, and won't learn the material well.
Homework is due at the beginning of the class (Wednesdays 7:00pm), to be uploaded on NYU classes.

The midterm and final exams will be closed-book, but you may bring one handwritten sheet of notes (8.5" $\times$ 11", both sides).

Mathematical Preliminaries: We will assume that students are fimiliar with the following concepts: $\sigma-$Algebras, probability measures, probabiity spaces, random variables, Martingales, convergence of random variables (in L^p, almost surely, in probability, and in law/distribution), Law of Large Numbers, Centrale Limit Theorem, charactersitic functions, Brownian motion and its properties, stochastic integrals w.r.t. Brownian motion (and a martingale more generaly), diffusions, Ito's lemma, SDEs, binomial models.

Tentative Schedule: (subject to changes)

- Week 1 (01/24), Monty: Stochastic Calculus review
  - Brownian motion, SDEs

- Link between PDEs and SDEs: Feynman-Kac, Fokker-Planck, Boundary value problem and exit times
  - Euler discretization and Monte-Carlo simulations of SDEs
  - Monte-Carlo simulations of correlated Gaussians via Cholesky decomposition

- Week 2 (01/31), Monty : Derivatives pricing I: Martingale approach
  - Girsanov
  - Martingale representation theorem
  - Risk Neutral pricing
  - Black Scholes formula and derivation
  - Fundamental Theorem of Asset Pricing
  - Dividend paying stock
  - Numerical aspects (Monte-Carlo)

- Week 3 (02/07), Monty : Derivatives Pricing II: PDE approach
  - Black-Scholes PDE
  - Numerical aspects of PDEs (implicit vs explicit Euler)
  - Interest Rate models (Hull-White, CIR)

- Week 4 (02/14), Monty : Term Structure Models
  - Affine Yield models (two factor Vasicek, Two-Factor CIR)
  - Heath-Jarrow-Morton model
  - Forward LIBOR model

- Week 5 (02/21), Bruno : Pricing and Hedging Financial derivatives: A practitioner's guide
  - Risk Neutral pricing
  - Pricing method
  - Hedging
  - Volatility

- Week 6 (02/28), Bruno : Volatility models I
  - A brief history of volatility models
  - Local Volatility Models
  - LVM implementation and calibration
  - Pricing with LVM

- Week 7 (03/07), Monty : xVA
  - Cox Default processes
  - Unilateral CVA
  - Unilateral DVA and bilateral CVA/DVA
  - Expected exposure
  - If time permits, Q&A session/general review before the midterm

- **Week 8 (03/21) : Midterm Exam**

- Week 9 (03/28), Bruno : Volatility Models II

  - Pricing with LVM II
  - Stochastic volatility models: Heston
  - Stochastic volatility models: SABR
- Week 10 (04/04), Guest Lecture : 'Special techniques for special events/Elections methodlogies'
- Week 11 (04/11), Monty : Introduction to Stochastic Optimal Control
  - Intro with an example
  - Dynamic programming principle and HJB equation
  - Verification theorem
  - Application: Merton's optimal investment/consumption
- Week 12 (04/18), Monty : Optimal stopping time problems and American options
  - Binomial tree setting
  - Discrete time theory
  - Continuous time theory
  - American options (call, put)
- Week 13 (04/25), Bruno : Functional Ito Calculus and volatility risk management
  - Functional Ito Calculus
  - Volatility Hedge
- Week 14 (05/02), Bruno : Conclusion
  - 10 Things to know as a Quant
  - Proper use of derivatives
  - If time permits, Q&A session
- **Week 15 (05/09) : Final Exam**