1 The Recitation on February 5, 2016

Key points: checking a function is a solution, dispersion relations, solving for a general solution, and others.

1. Problem 1.7 in EPDE on Page 9.
2. Problem 1.6 in EPDE on Page 9.
4. Dispersion relation of the damped wave equation $u_{tt} + u_t = u_{xx}$.
5. Starting from Cauchy-Riemann equation, we showed that real and imaginary parts of complex analytic functions are harmonic functions. Then comes an example $u(x, y) = \text{Re}(z^3)$, the monkey saddle.
7. A general solution of $u_{xt} + 3u_x = 1$.

2 The Recitation on February 12, 2016

Key points: first order PDE, and methods of characteristics.

1. Solution of (1.5) in §1.2 of APDE on Page 16.
2. Example 1.9 in §1.2 of APDE on Page 17.
3. Example 1.11 in §1.2 of APDE on Page 18.
5. Problem 12 in §1.2 of APDE on Page 27.

3 The Recitation on February 19, 2016

Key points: revisit of what was discussed in the last recitation (first order PDE and methods of characteristics), classification of second order PDE, and solving a second order hyperbolic equation with generic coefficients.

1. Example 1.9 in §1.2 of APDE on Page 17.
2. Example 1.11 in §1.2 of APDE on Page 18.
4. Problem 4.9 in EPDE on Page 54.
4 The Recitation on February 26, 2016

Key points: more about methods of characteristics, and solution of heat/diffusion equation.

1. Problem 13 in §1.2 of APDE on Page 27 ($f(x, t)$ instead of $f(x)$ is put on RHS).
2. Characteristics of the inviscid Burgers' equation $u_t + uu_x = 0$.
3. Two ways to get solution of heat equation with initial data $u(x, t = 0) = \frac{1}{\sqrt{\pi}} \exp(-(x - 2)^2)$ — brute-force computation and space-time shift.

5 The Recitation on March 4, 2016

Key points: a boundary value problem of wave equation, Duhamel’s formula, the $\delta$-function.

1. Problem 4.7 in EPDE on Page 54 (also in HW 4).
2. Problem 4 in §2.5 of APDE on Page 106, and a modification of that: $u_t + 2tu_x = xe^{-t}$. Note that the original problem is translation invariant in $t$, but the modified problem is not. As a result, when applying the Duhamel’s formula, the forward map should be treated carefully in the latter case. The problem can also be solved by integration along the characteristics.
3. Some remarks on the $\delta$-function:
   (a) It is best to understand the $\delta$-function as a linear functional. We always define a functional by characterizing its effect on good test functions (to be more precise, $C_0^\infty$-functions). For example, any function $f$ in the classic sense can be realized as a functional in a natural way, i.e.,

   $$ L_f(g) \triangleq \int g(x) dx, \quad \forall g \in C_0^\infty(R). $$

   The definition of the $\delta$-function is given by

   $$ \delta[g] = L_\delta(g) \triangleq g(0), \quad \forall g \in C_0^\infty(R). $$

   (b) Formal integral representations like $g(0) = \int g(x) dx$ are helpful in understanding why the definitions concerning generalized functions are natural (e.g. recall how we define $\delta'$), but they are just formal.

   (c) Convergence of a sequence of functions $\{f_n\}$ to the $\delta$-function is defined as follows: we say $f_n \to \delta$ (in the sense of distribution) as $n \to +\infty$ if

   $$ \lim_{n \to +\infty} \int f_n(x) g(x) dx = g(0), \quad \forall g \in C_0^\infty(R), $$

   or equivalently, if

   $$ \lim_{n \to +\infty} L_{f_n}(g) = L_\delta(g), \quad \forall g \in C_0^\infty(R). $$