

Homework 8 [40 pts]

ODE Fall 2012

A. Poner

[25 pts]

① (a) Is the fixed point

[10 pts] $\theta = \pi$

a stable or unstable point for the damped pendulum equation

$$\theta'' + \gamma \theta' + \omega^2 \sin \theta = 0$$

where $\gamma > 0$ is a friction constant and $\omega^2 > 0$?

② (b) If $\gamma = 0$ (undamped),

[7.5 pts]

is $\theta = \pi$ stable or unstable? Classify the type of fixed point it is (node, saddle, spiral, center, etc.).

③ (c) How about $\theta = 0$ for the case $\gamma = 0$?

[7.5 pts]

2/

(2)

[15 pts]

Consider the fixed point

$$y_1 = y_2 = 0$$

for the system of ODEs

$$\begin{cases} y_1' = -y_1 - y_2 \\ y_2' = y_1 - y_2^3 \end{cases}$$

using the Lyapunov function

$$V(y_1, y_2) = y_1^2 + y_2^2$$

and see whether the fixed point is stable, asymptotically stable, or something else.