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Homework 7 [60 pts]

ODE Fall 2012

A. DONE

①

3x5 pts  
= [15 pts]

For the three systems of ODEs from problems 1, 3 and 4 in Homework 6, classify the fixed point  $y_0 = 0$  as one of proper/improper ~~node~~ node, saddle point, spiral point or center, and determine whether the origin  $y_0 = 0$  is a stable or unstable point.

If it is stable, determine whether it is asymptotically stable. To repeat the equations:

①

$$A = \begin{bmatrix} 5 & -2 \\ 4 & -1 \end{bmatrix}$$

②

$$A = \begin{bmatrix} 3 & 5 \\ -5 & 3 \end{bmatrix}$$

③

$$A = \begin{bmatrix} 2 & -3 \\ 3 & -4 \end{bmatrix}$$

②

[30 pts] For each of the following scalar ODEs, decide whether the origin  $y=0$  is a stable, asymptotically stable, or unstable point:

5 pts

(a)  $y' = y$

10 pts

(b)  $y' = -y^2$

7.5 pts

(c)  $y'' + 4y' + 4y = 0$

7.5 pts

(d)  $y'' + 2ky' + \alpha^2 y = 0$

where  $k > 0$  and  $\alpha^2 > 0$  are given real constants.

③

Show that  $y=0$  is an unstable solution of the system

[15 pts]

$$y' = \begin{bmatrix} -1 & e^{2t} \\ 0 & -1 \end{bmatrix} y$$