System Support for Scientific Computation

Notes by David Bindel from a lecture of W. Kahan

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1 Intransitivity of correctness/accuracy

Functions : $f(x), g(x)$ s.t. $h(x) = f(g(x))$

Programs : $F(X), G(X)$ s.t. $H(X) = F(G(X))$

Assume $F$ and $G$ are correctly rounded.

1.1 Exercise for the student

Find examples where $H$ bares no resemblance to $h$.

Can also get the right answer for the wrong reason:

$F,G$ somewhat inaccurate, $H$ very accurate.

Transitivity of accuracy does succeed often enough that it’s worth fighting for. How do we understand where to fight?

Example from when Kahan was at Toronto: program crashed because the computed square root failed to be monotone. (It then also turned out that log was off, exp was sloppy, etc.)

Side Note: Aircraft wings are actually twisted some, so that if stall occurs, it affects the shoulder first, rather than affecting the tip. where the moment could flip the plane.

(c. 1960)

After rewriting log, Kahan re-ran tapes of the previous days jobs that used log (the OS was instrumented then to track library calls, in the event that someday, library users might be charged royalties).

It turned out that some simulations of a blown wing changed dramatically. The program had produced a curve which dropped too early on the 7090 in single precision (due to poor log implementation) and in double (due to lack of a guard digit). Replacing

$x - 1.0$

with

$(x - 0.5) - 0.5$
fixed the latter problem.  
(c. 1969) Kahan came to Berkeley. Brought along a wonderful ODE solver: had an RK routine and a VSVO) multistep method. It broke on the CDC 6600 here. The reason? compare was a random function of the trailing word when using software simulated double.

Other perversities on that same machine:

\[
\frac{nz}{nz} = 0 \text{ (or diagnostic)} \\
\frac{nz}{(\text{> 1})} = 0 
\]

Niklaus Wirth got a copy of the report detailing these anomalies, and sent a howling letter to CDC, who replied: "Our computers don't do that, and if they did, it wouldn't matter."

Question: Why is this reasoning flawed, and why/how should it be done differently?

In the 20s, some used 6-7 digit tables for exterior ballistics. With the 30s, WPA sheltered mathematicians used tables to 8 digits.

People started to want more figures than obvious for various reasons: stiff structures, regressions based on correlated parameters, etc.

But the guys with money built bombs. The didn't care as much about rounding, and they called the shots.

2 Triumph of backward analysis

Turing ’49, mentioned the idea in a throwaway comment in a paper refuting an error analysis by Von Neumann of Gaussian elimination. Wilkinson noticed later. Givens ’52, mentioned the notation in a technical report that never was published in another format.

Von Neumann’s view: Floating point is silly. If you understand your problem, you can scale the variables so you can use fixed point.

Kahan: "There is something about numerical computation that brings out the urge to gamble."

Lancester (famous for his mathematical theory of warfare) in a paper on the stresses on the old Aswan Dam, noted the cost of every correct decimal digit was about the same as the cost of a loaf of bread.

The cost of computation now is far below its value, so that the economics of the situation have changed. With the advent of commodity computing, error analysis costs much more than the computation.

The big cost now is in getting to run time. Who can afford that? Well, the federal govt...

3 Interval arithmetic - 1960s

Basically works well when you can form the problem as a fixed point calculation of a contractive map. But this only works well when there is a single fixed point.

4 Significance arithmetic - Metropolis

We'll look at this and see why it doesn't work.
5 Exercise

What error rate (assuming only 1GFLOP machine would be required so that someone need only be bothered perhaps once a month?