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Algebraic approaches to random matrix theory and combinatorial problems.

A program of study done with Peter Forrester has been completed which has explicitly evaluated a generalisation of the traditional spectral statistic in random matrix theory, the probability that a given spectral interval  $J \subset I$  is free of eigenvalues, for all the finite unitary ensembles with classical weights and all the scaled limits in terms of Jimbo-Miwa-Okamoto  $\tau$ -functions. This generalisation is the average with respect to the eigenvalue PDF for the Gaussian, Laguerre and Jacobi unitary ensembles,

$$\tilde{E}_N(J; \mu; \xi) := \left\langle \prod_{l=1}^N [1 - \xi \chi_{I \setminus J}(\lambda_l)] (\lambda - \lambda_l)^\mu \right\rangle$$

where  $\chi_J(\lambda_l) = 1$  for  $\lambda_l \in J$  and  $\chi_J(\lambda_l) = 0$  otherwise,  $\mu, \xi \in \mathbb{C}$  and  $\lambda \in \partial J$ , which includes both the gap probability and moments of the characteristic polynomial as special cases, and allows the parameters in the theory to fully match those of the Painlevé theory. An explicit global picture of this will be presented in which classical solutions of  $P_{VI}$ ,  $P_V$  and  $P_{IV}$  naturally appear in the finite ensembles, however the scaled ensembles are transcendental solutions of the Painlevé equations and generalise the bulk, hard edge and soft edge universality classes to the general  $P_V$   $\tau$ -function with three arbitrary parameters, to the general  $P_{III}$  with two parameters and the general  $P_{II}$  with one respectively. The boundary conditions imposed on the differential equations for the  $\sigma$ -forms in scaled ensembles lead to a specific class of transcendental solutions that have not been studied much - the separatrix solutions. Many types of difference equations for the  $\sigma$ -form with respect to any of the independent parameters will be presented, these being equivalent to the discrete Painlevé equations, examples of duality relations between the discrete parameters and the continuous ones restricted to the natural numbers will be given, and the significance of special identities for the densities and distribution functions of the extremal eigenvalues and spacings will be discussed.