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A geometric theory of nonlinear morphoelastic shells

Abstract

Various theories of shells have been used to model biological membranes or membranes with defects. Physicists typically favour Helfrich-type models that assume that the membrane is fluid along its surface but resists bending. Mechanicians typically use incompatible nonlinear shell theories. Here we revisit these different theories and formulate a general geometric theory of nonlinear morphoelastic shells that can model either the time evolution of residual stresses induced by bulk growth or distributed defects in two-dimensional bodies. In this geometric theory, growth or defects are modeled using an evolving referential configuration for the shell. We consider the evolution of both the first and second fundamental forms in the material manifold by considering them as dynamical variables in the variational problem. Their evolution can be used to model both surface growth, defects, and remodeling (the evolution of spontaneous curvatures). Then, we use a Lagrangian field theory to derive the governing equations of motion. We also consider the particular case where defects can be modeled by a Rayleigh potential. This talk is based on joint work with Souhayl Sadik, Arzhang Angoshtari, and Arash Yavari.