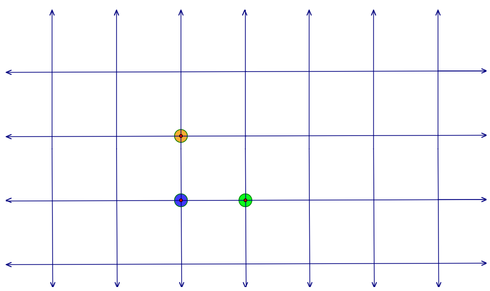
Leap Frog Activity

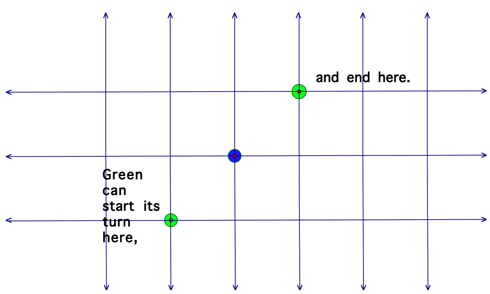
"Leap frog" may be done with a class as an individual activity, or as a game with students taking turns.

This activity requires a grid and works well on a large floor with square tiles. Begin by putting three counters on three vertices of one of the tiles. These counters are frogs, and they move by playing 'leapfrog.' To take a turn, a student moves his counter to another vertex by "reflecting" his frog over another frog. That is, frog A can move to a point for which any other frog B sits at the midpoint between frog A's old and new positions. The job of the group is to get any one of the three frogs to land on the fourth vertex of the original square (which can be marked with an x made out of tape).

This is the starting position:



In this example, the green frog jumps over the blue frog to a new position:



In fact, it is not possible to move a frog to the winning vertex. If we introduce coordinates on the lattice so that the frogs start at (0,0), (0,1), and (1,0), we find that the parity of the frogs' coordinates never changes. And since none of the frogs starts at a point with two odd coordinates, he can never land on (1,1).

Students can find this out by keeping track of those places where frogs do land. They will find out that the forbidden vertices form a sub-lattice, the lattice of points with two odd coordinates. It takes some experimentation to understand this, and the teacher must decide based on the students' level of understanding whether to explain the solution in terms of the invariant parity of the coordinates.

It is perfectly acceptable to run this activity without reaching the conclusion that it is impossible to get to the fourth vertex. Students will gain intuitions about midpoints, reflection in a point, and the slope of a line in trying to get a frog onto the fourth vertex.

As an extension, students can notice that the area of the triangle formed by the positions of the three frogs is also invariant. The area is always ½ square unit.