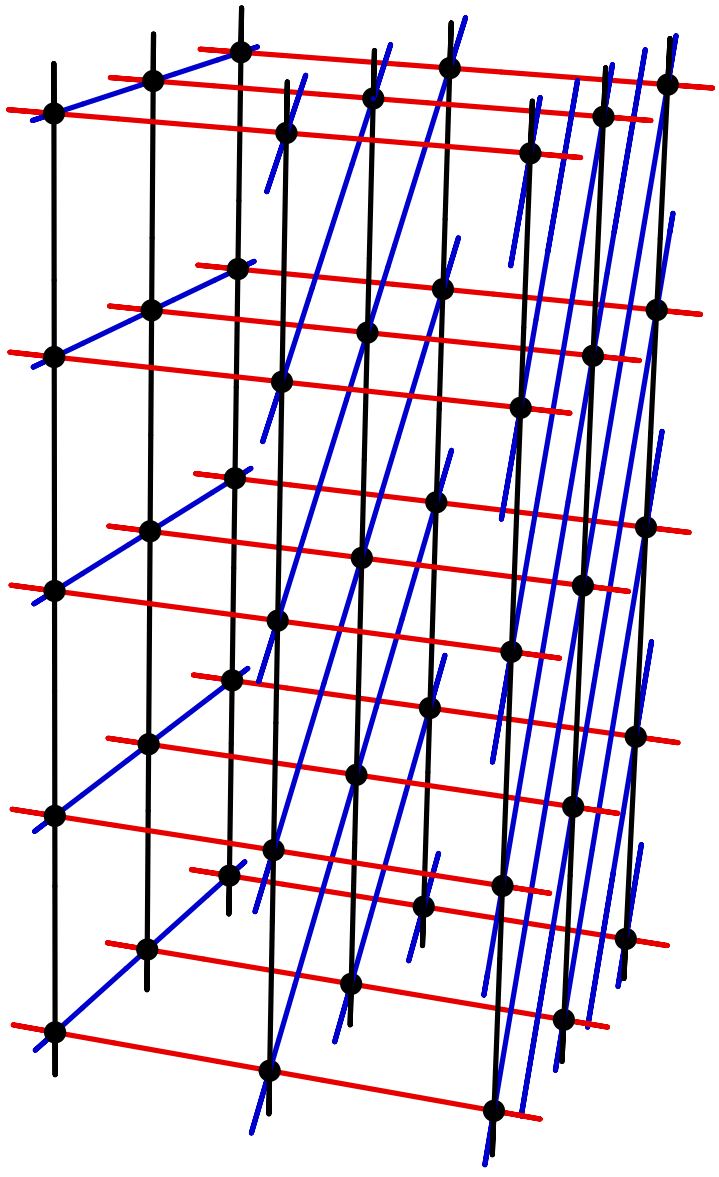
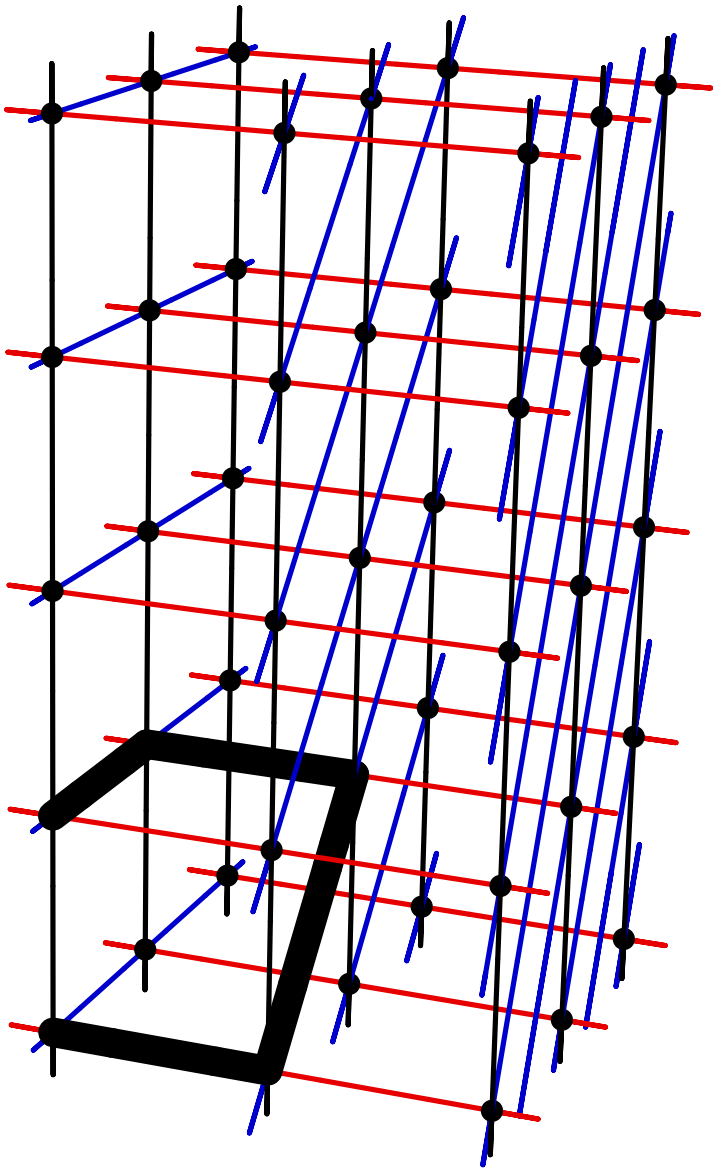
The Cayley graph of $H^3_{\mathbb{Z}}$

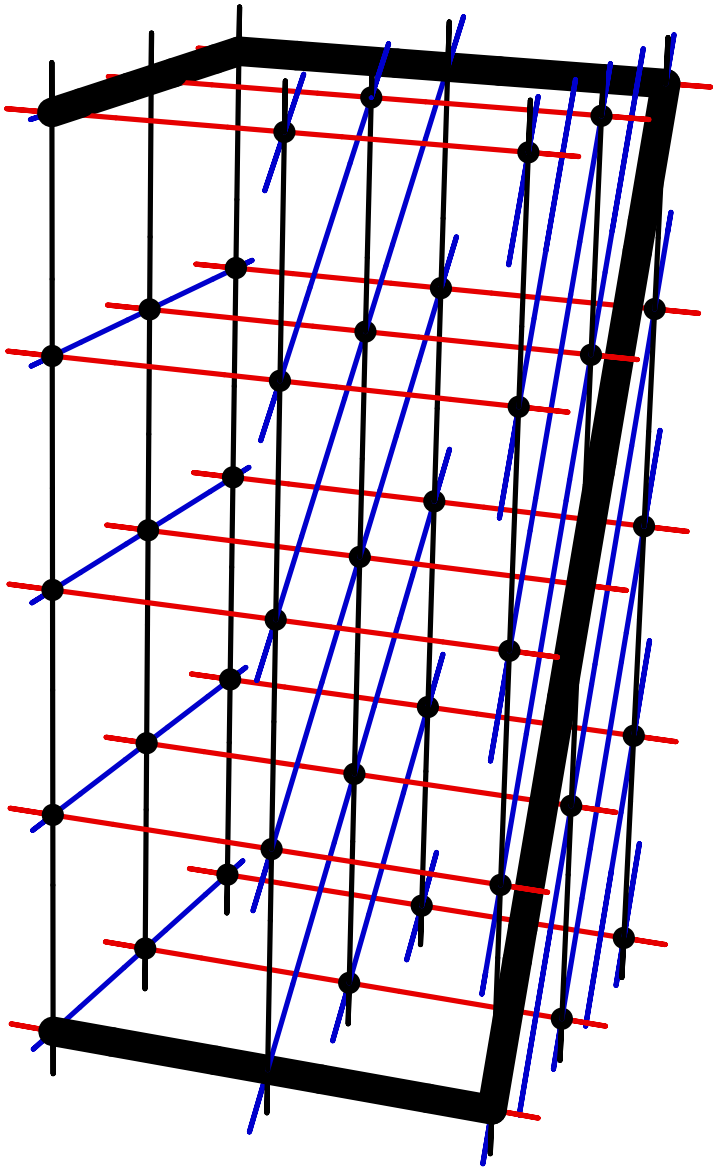


The Cayley graph of $H^3_{\mathbb{Z}}$



$$z = xyx^{-1}y^{-1}$$

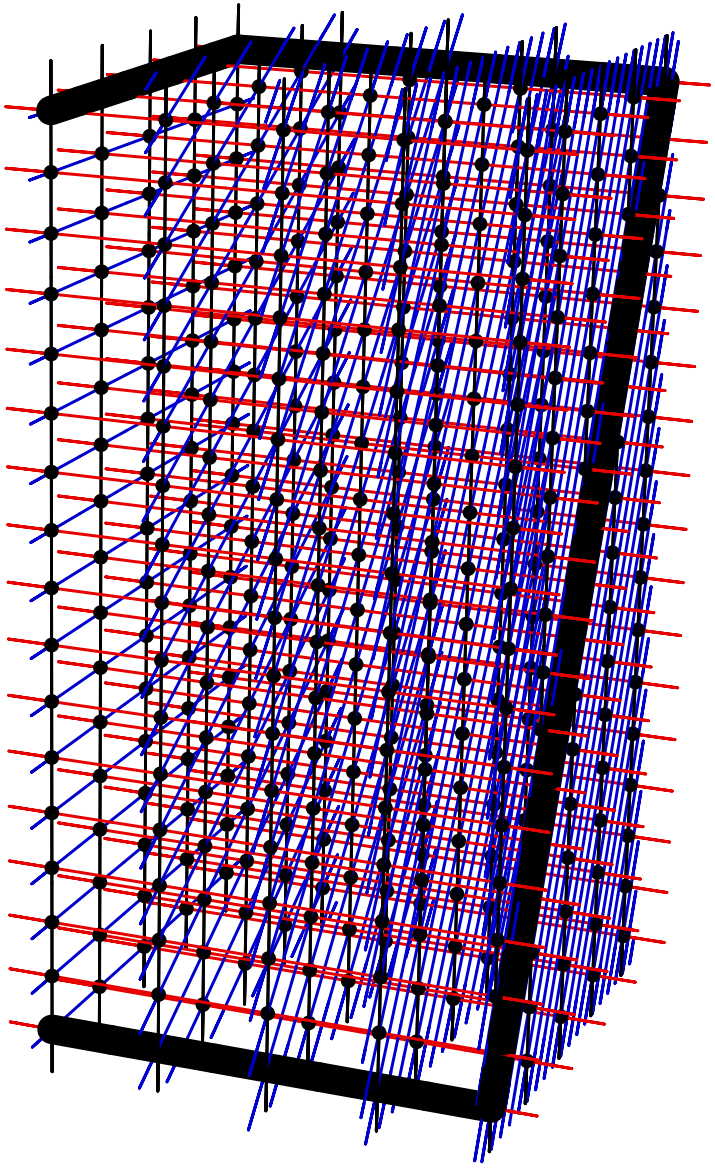
The Cayley graph of $H^3_{\mathbb{Z}}$



$$z = xyx^{-1}y^{-1}$$

$$z^4 = x^2y^2x^{-2}y^{-2}$$

The Cayley graph of $H^3_{\mathbb{Z}}$

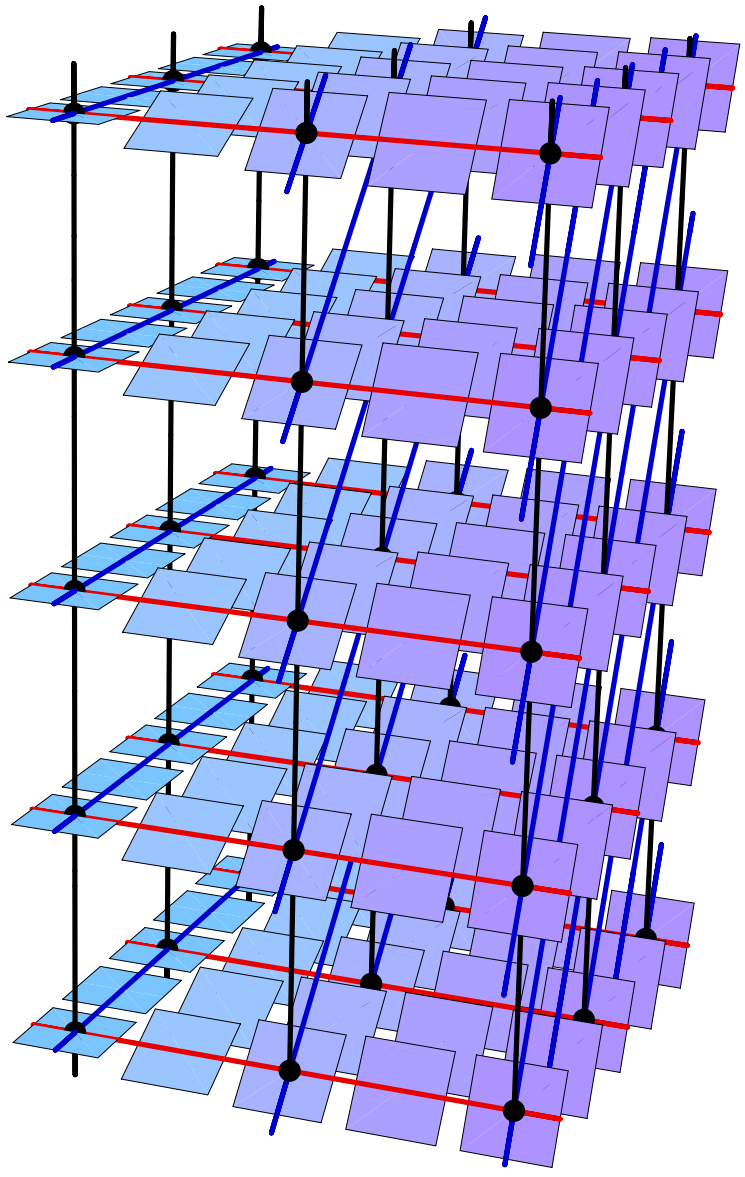


$$z = xyx^{-1}y^{-1}$$

$$z^4 = x^2y^2x^{-2}y^{-2}$$

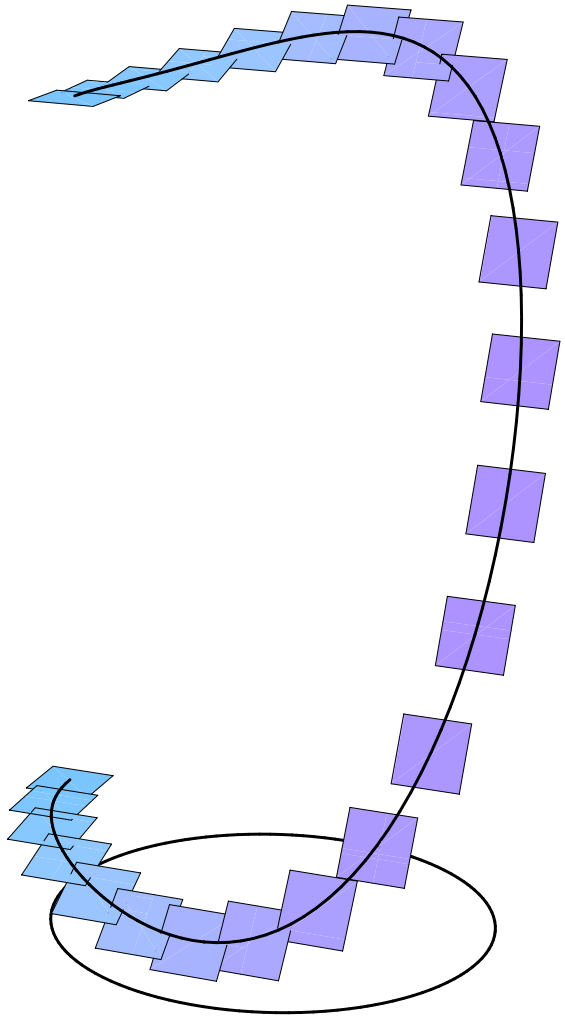
$$z^{n^2} = x^ny^nx^{-n}y^{-n}$$

From Cayley graph to sub-riemannian metric



Horizontal planes spanned by red and blue edges.

Horizontal curves in H



A geodesic in H